Divergenz: Aufgaben 8-15

Bestimmen Sie die Divergenz folgender Vektorfelder.

Aufgabe 8: $\vec{F} = -\sin(2\,y) \cdot \vec{i} + \cos(2\,x)\,\vec{j}$

Aufgabe 9: $\vec{F} = \cos x \, \sin y \cdot \vec{i} - \sin x \, \cos y \cdot \vec{j}$

Aufgabe 10: $\vec{F} = \cos x \, \sin y \cdot \vec{i} + \sin x \, \cos y \cdot \vec{j}$

Aufgabe 11: $\vec{F} = -\cos x \cdot \vec{i} + \sin y \cdot \vec{j}$

Aufgabe 12: $\vec{F} = -x\,y\,\vec{i} + y^2 \cdot \vec{j}$

Aufgabe 13: $\vec{F} = -2\,x\,y\,\vec{i} + y^2 \cdot \vec{j}$

Aufgabe 14: $\vec{F} = x\,y^2\,\vec{i}$

Aufgabe 15: $\vec{F} = x\,y^2\,\vec{i} + x^2\,y\,\vec{j}$

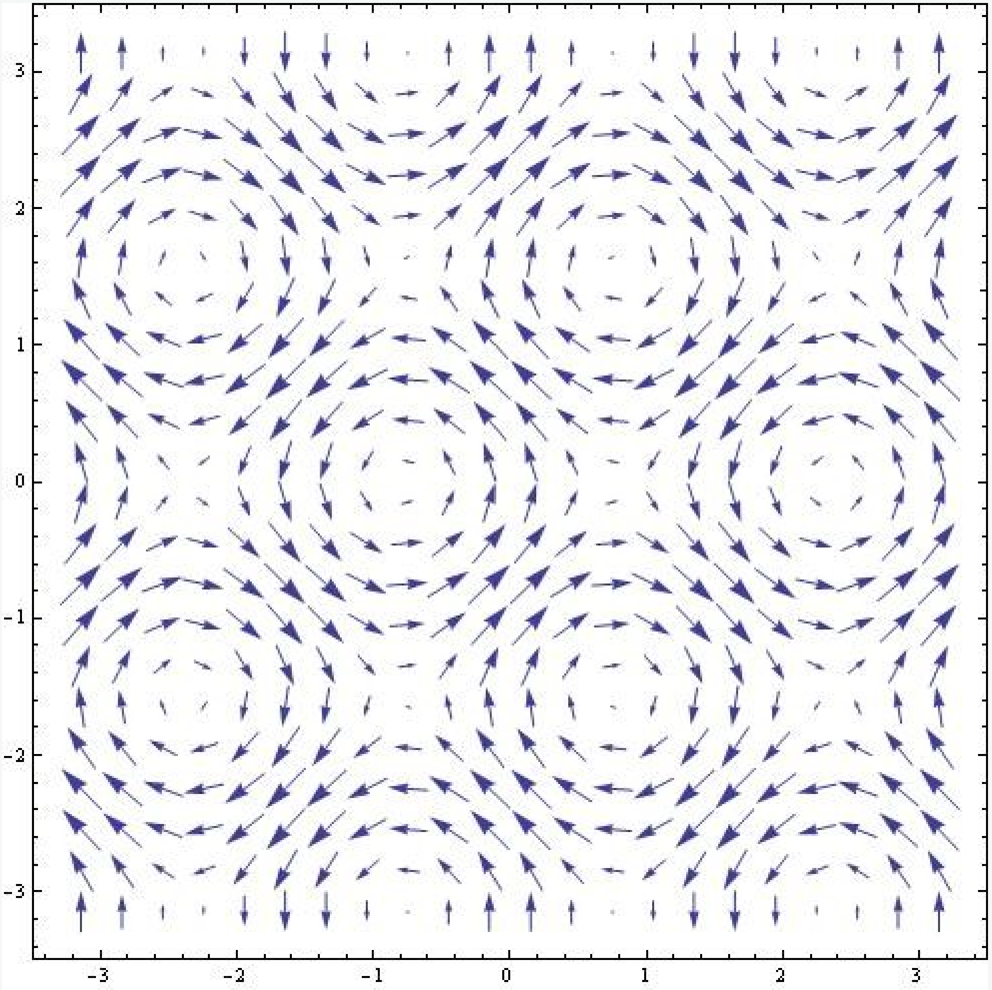


*Abb. L8-1: Das Vektorfeld der Funktion **F** (x, y)*

$$\vec{F} = -\sin(2\,y) \cdot \vec{i} + \cos(2\,x)\,\vec{j}\,, \qquad div\,\vec{F} = 0$$

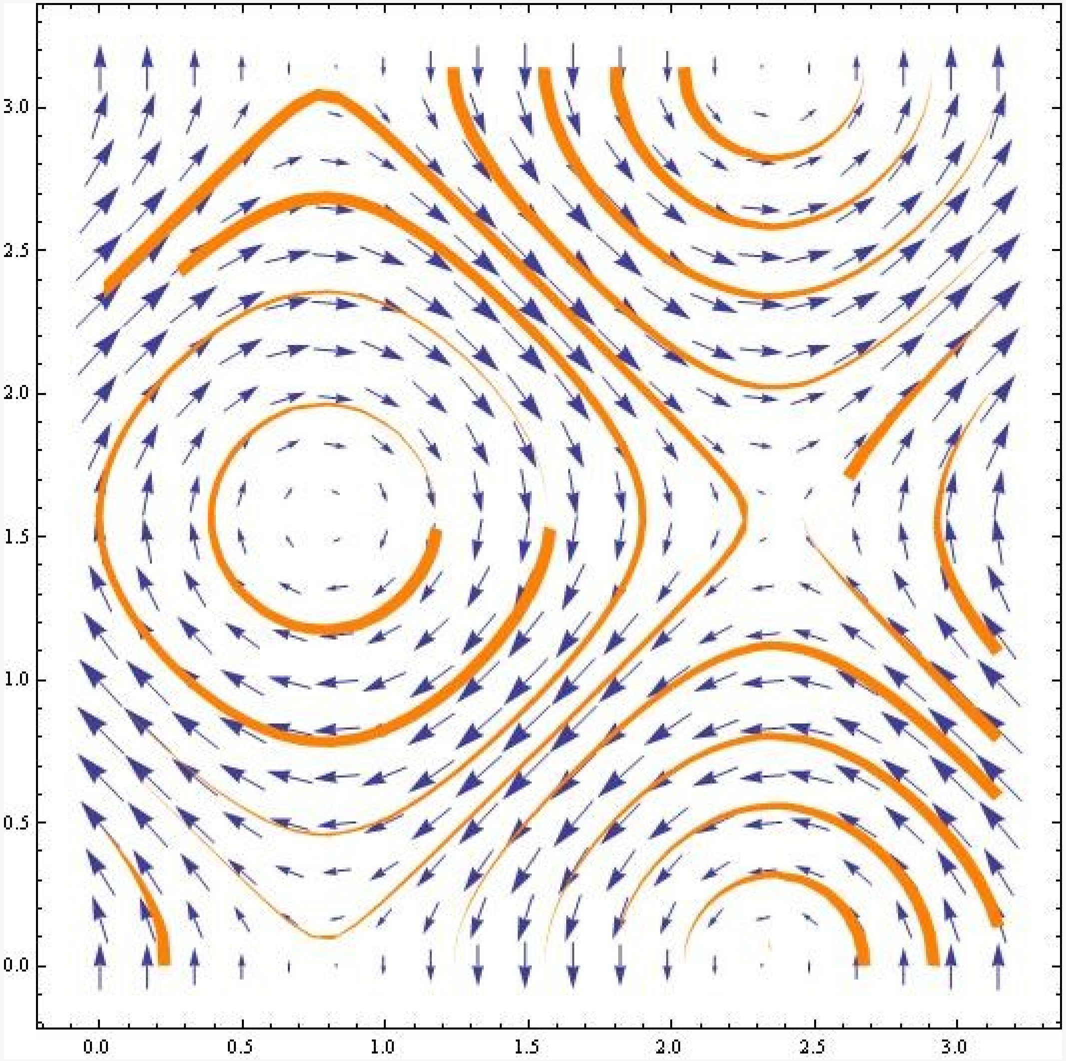


Abb. L8-2: Das Vektorfeld und Feldlinien der Funktion $\boldsymbol{F}\,(x, y)$

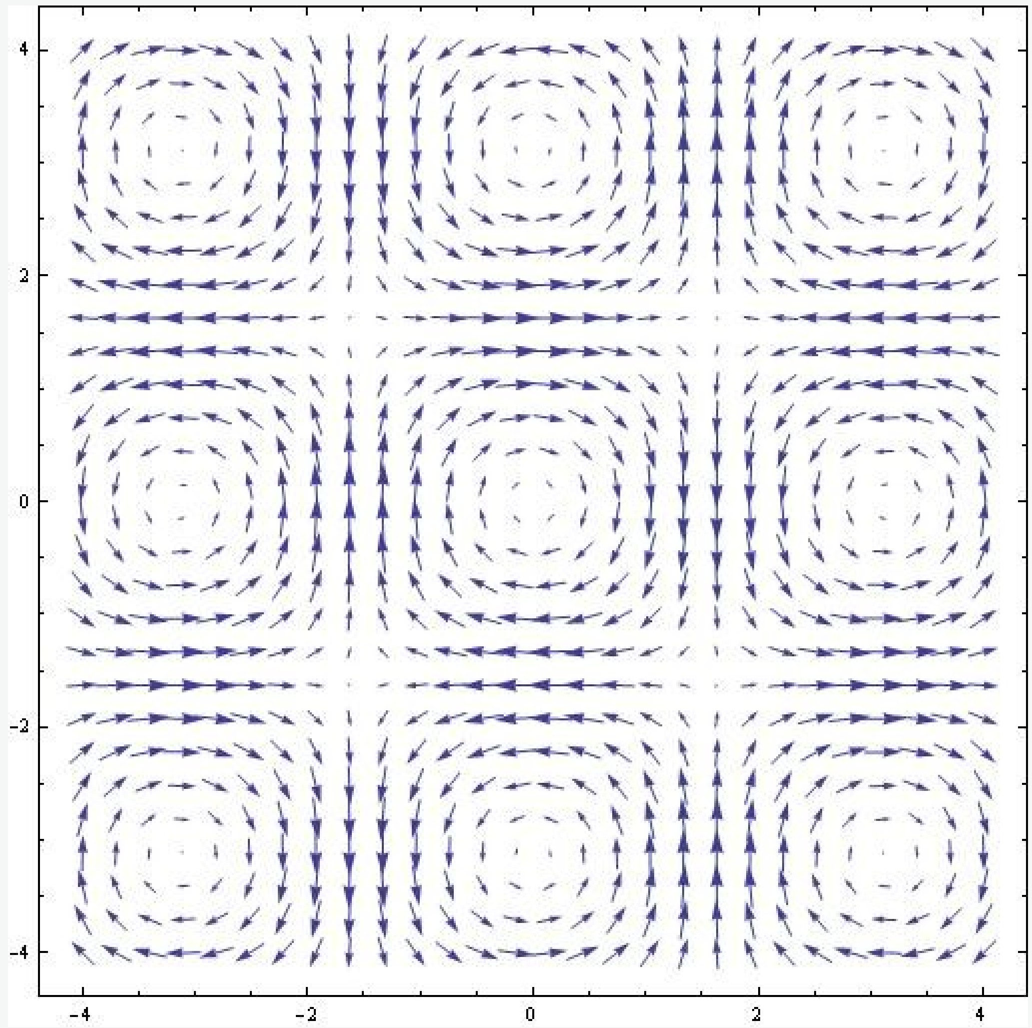


*Abb. L9-1: Das Vektorfeld der Funktion **F** (x, y)*

$$\vec{F} = \cos x \ \sin y \cdot \vec{i} - \sin x \ \cos y \cdot \vec{j} \,, \qquad div\, \vec{F} = 0$$

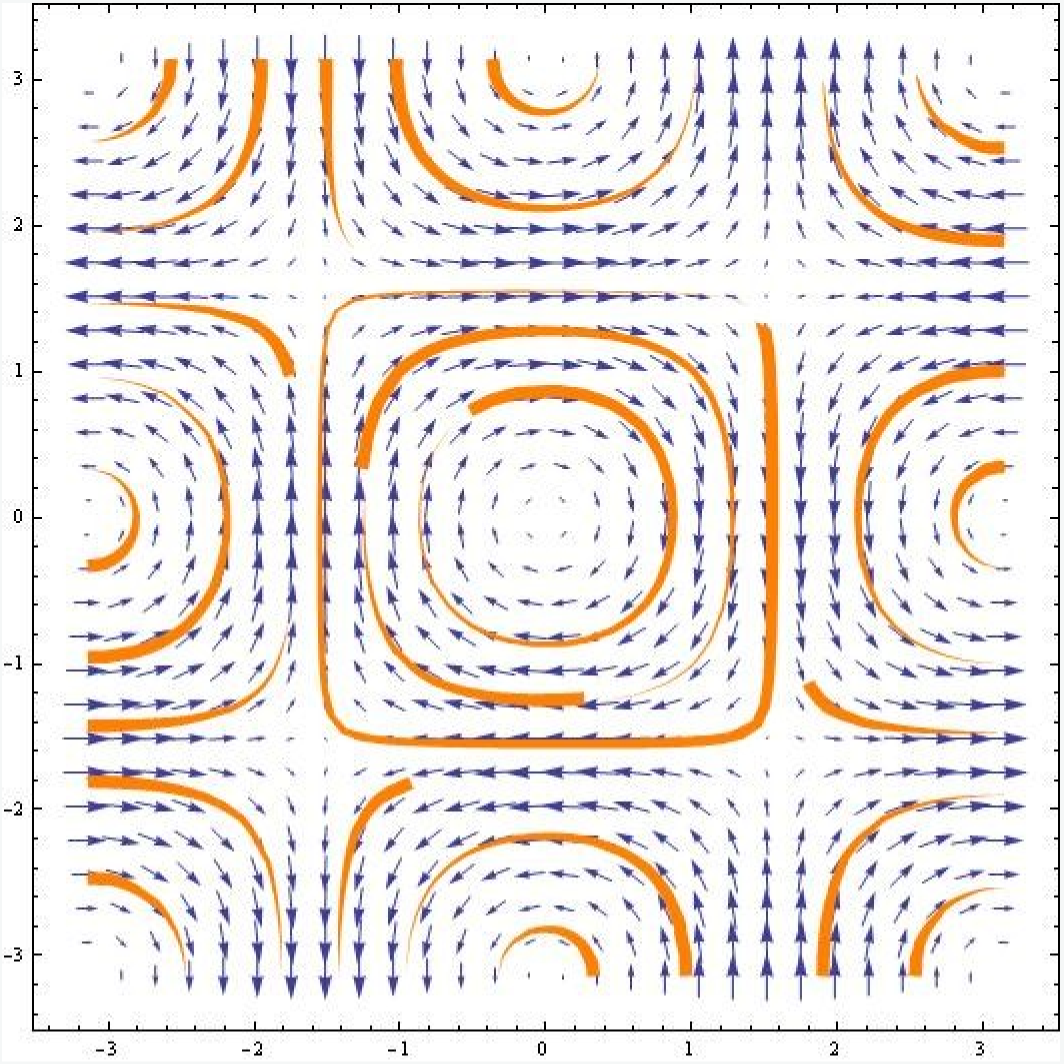


Abb. L9-2: Das Vektorfeld und Feldlinien der Funktion ***F*** *(x, y)*

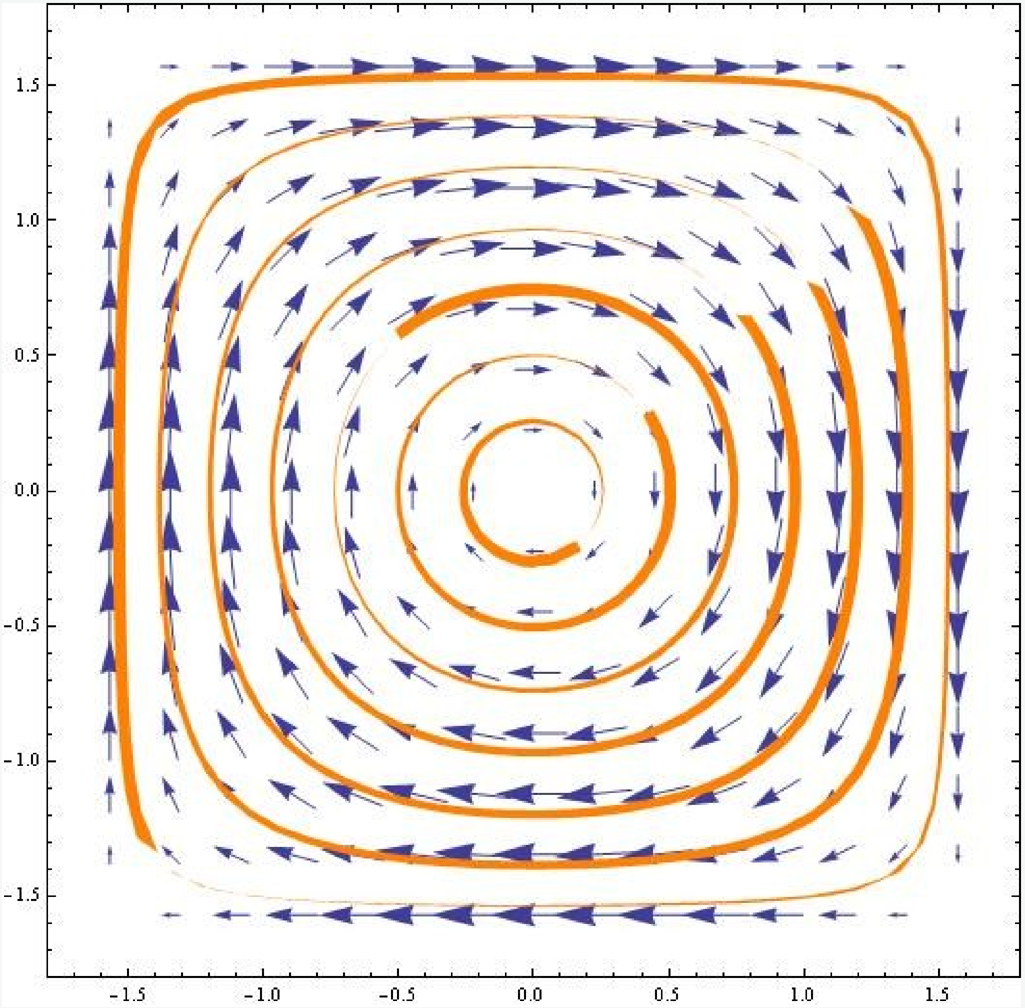


*Abb. L9-3: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

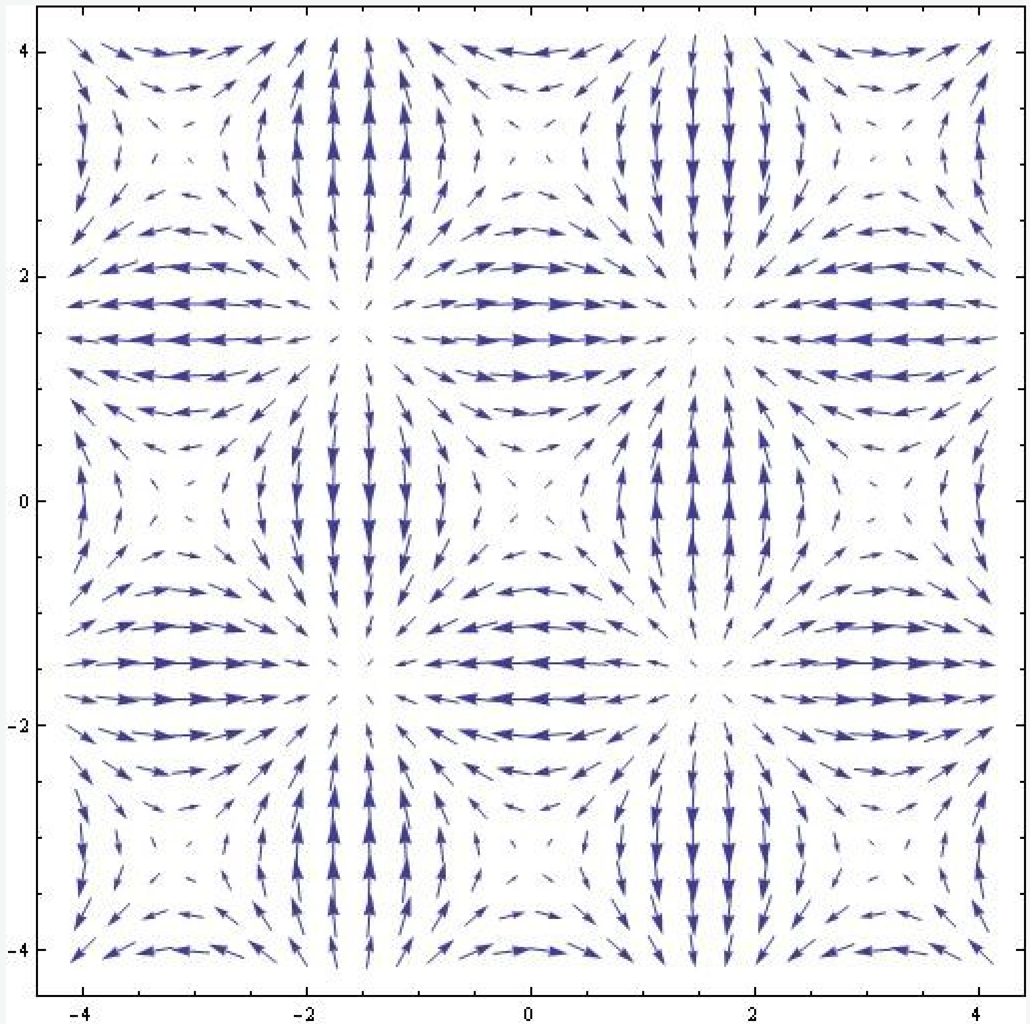


*Abb. L10-1: Das Vektorfeld der Funktion **F** (x, y)*

$$\vec{F} = \cos x \ \sin y \cdot \vec{i} + \sin x \ \cos y \cdot \vec{j} \, , \qquad div \, \vec{F} = -2 \sin x \ \sin y$$

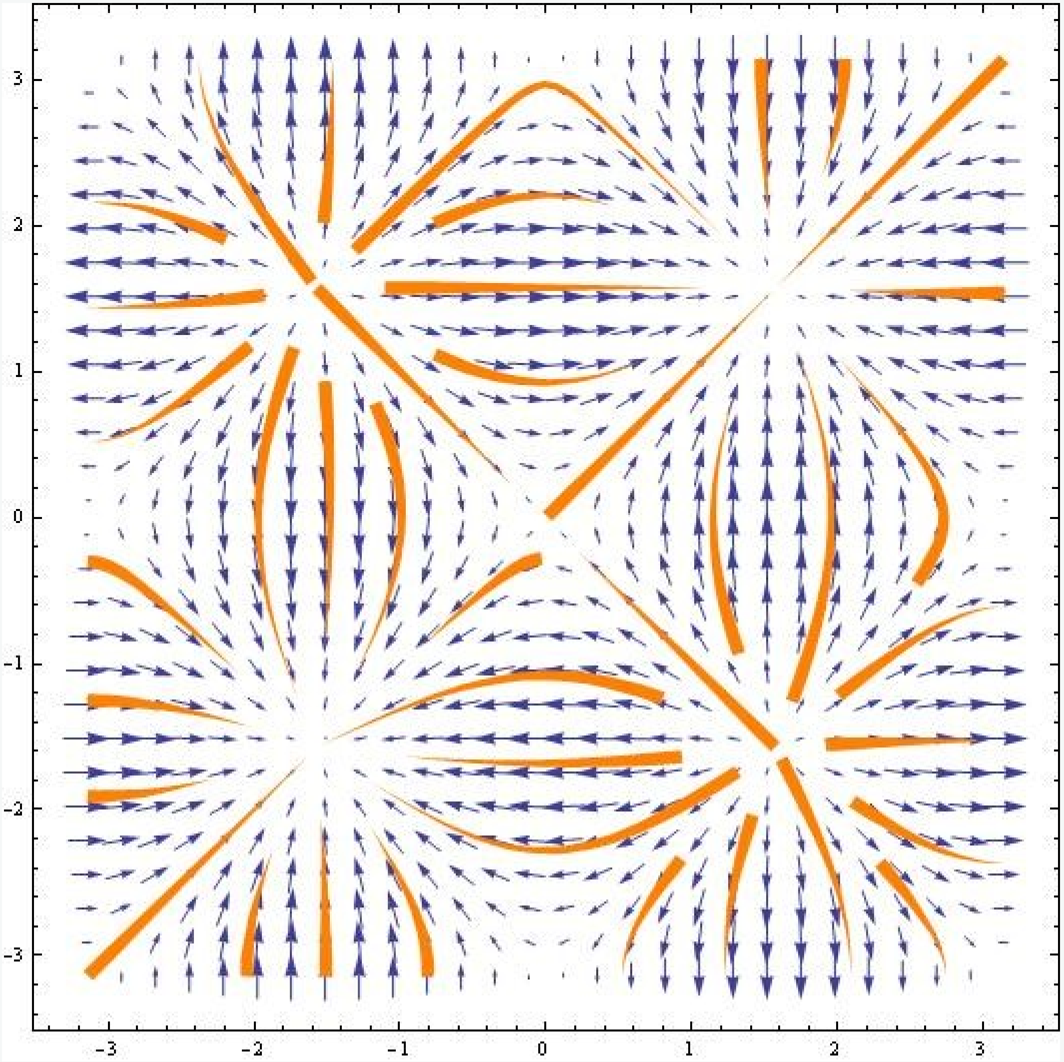


Abb. L10-2: Das Vektorfeld und Feldlinien der Funktion ***F*** *(x, y)*

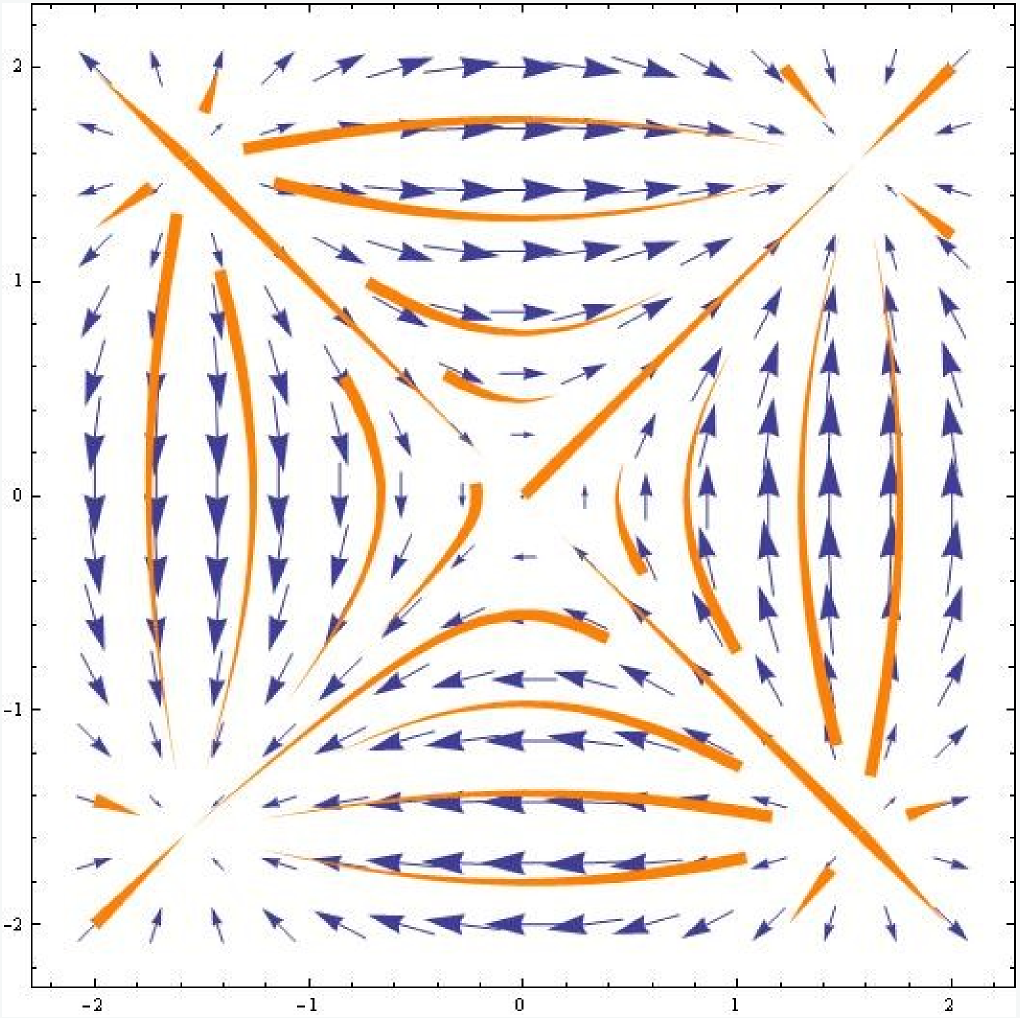


*Abb. L10-3: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

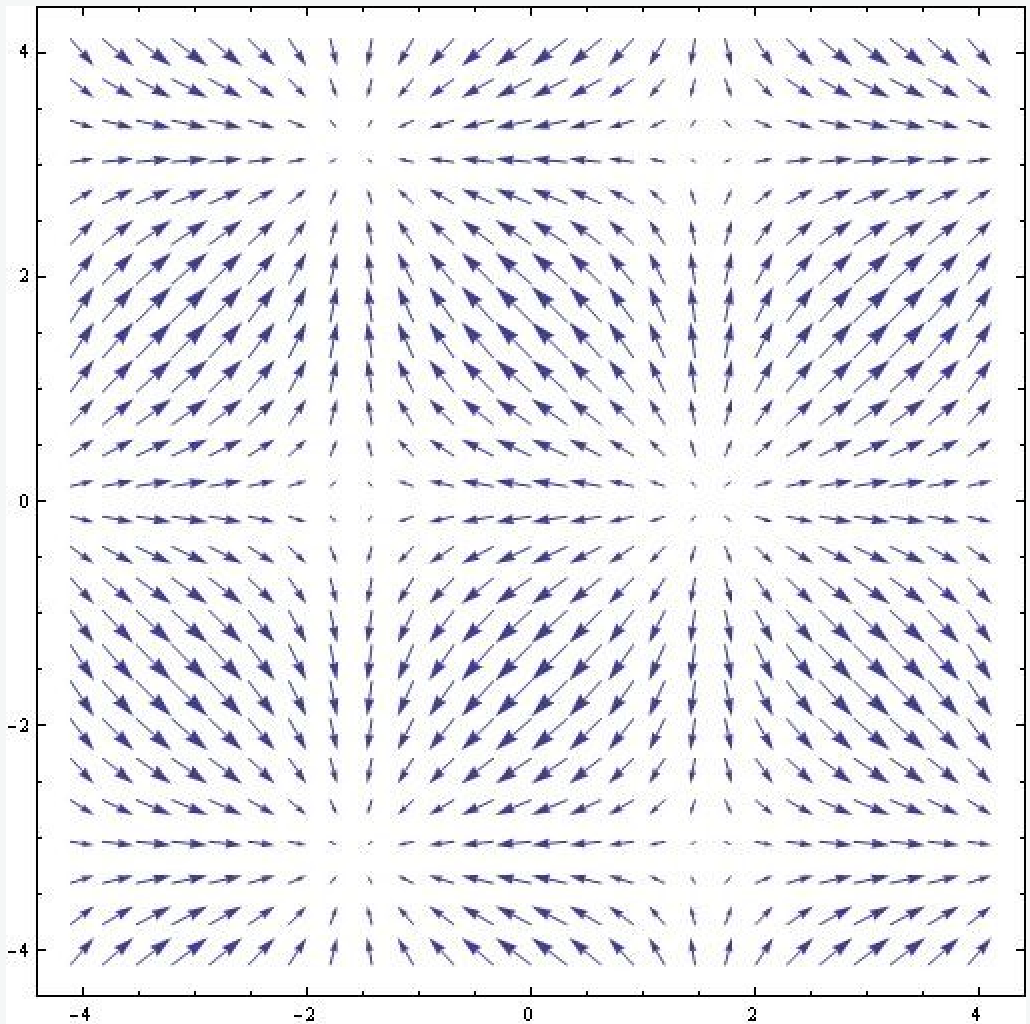


*Abb. L11-1: Das Vektorfeld der Funktion **F** (x, y)*

$$\vec{F} = -\cos x \cdot \vec{i} + \sin y \cdot \vec{j}\,, \qquad div\,\vec{F} = \sin x + \cos y$$

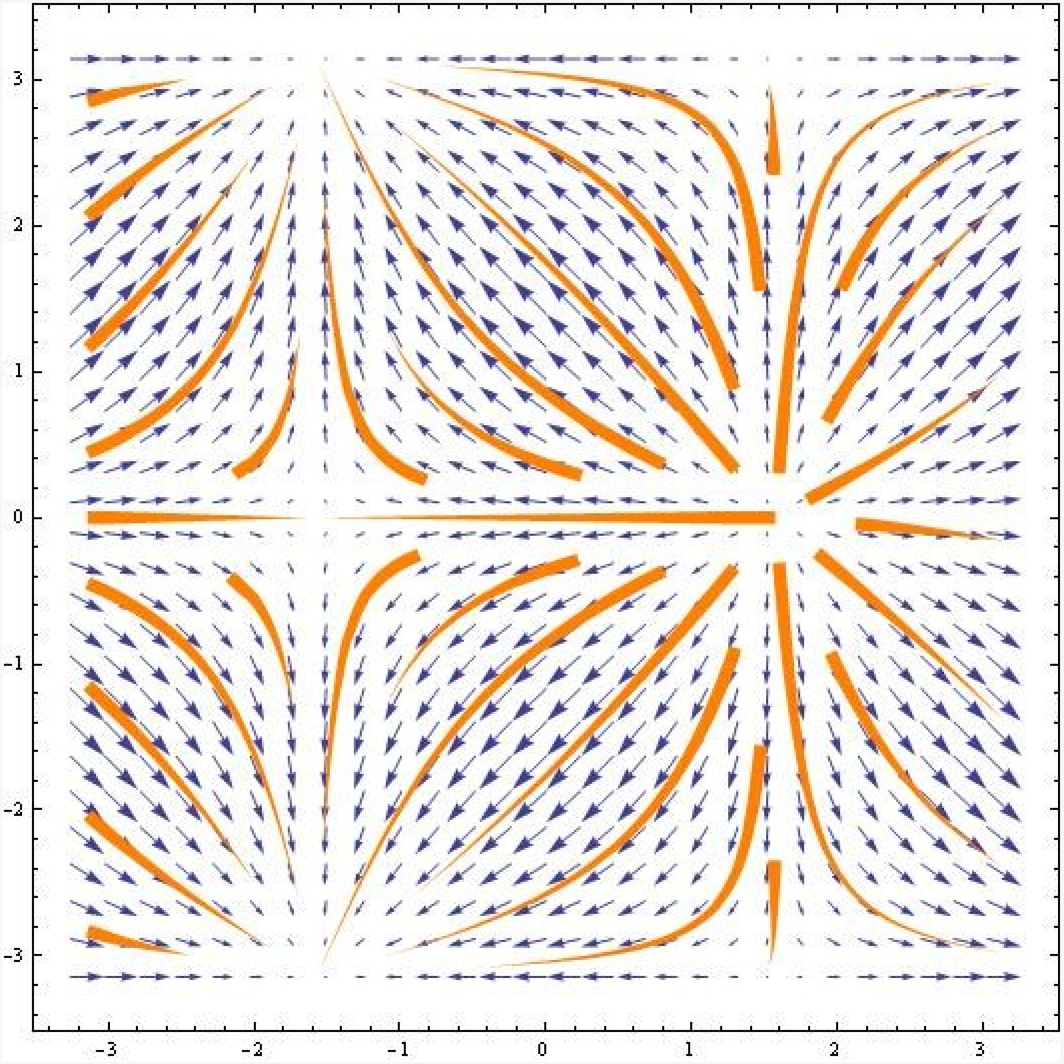


Abb. L11-2: Das Vektorfeld und Feldlinien der Funktion ***F*** *(x, y)*

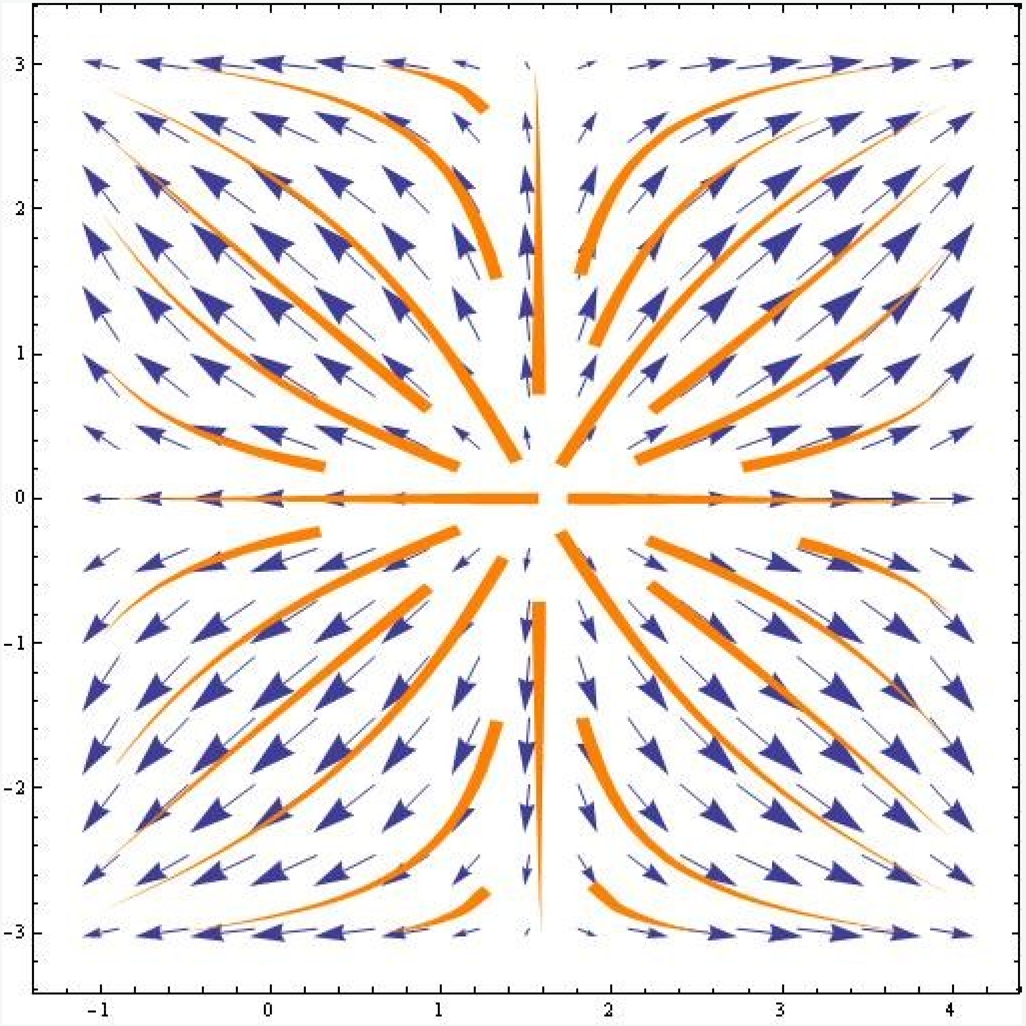


*Abb. L11-3: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

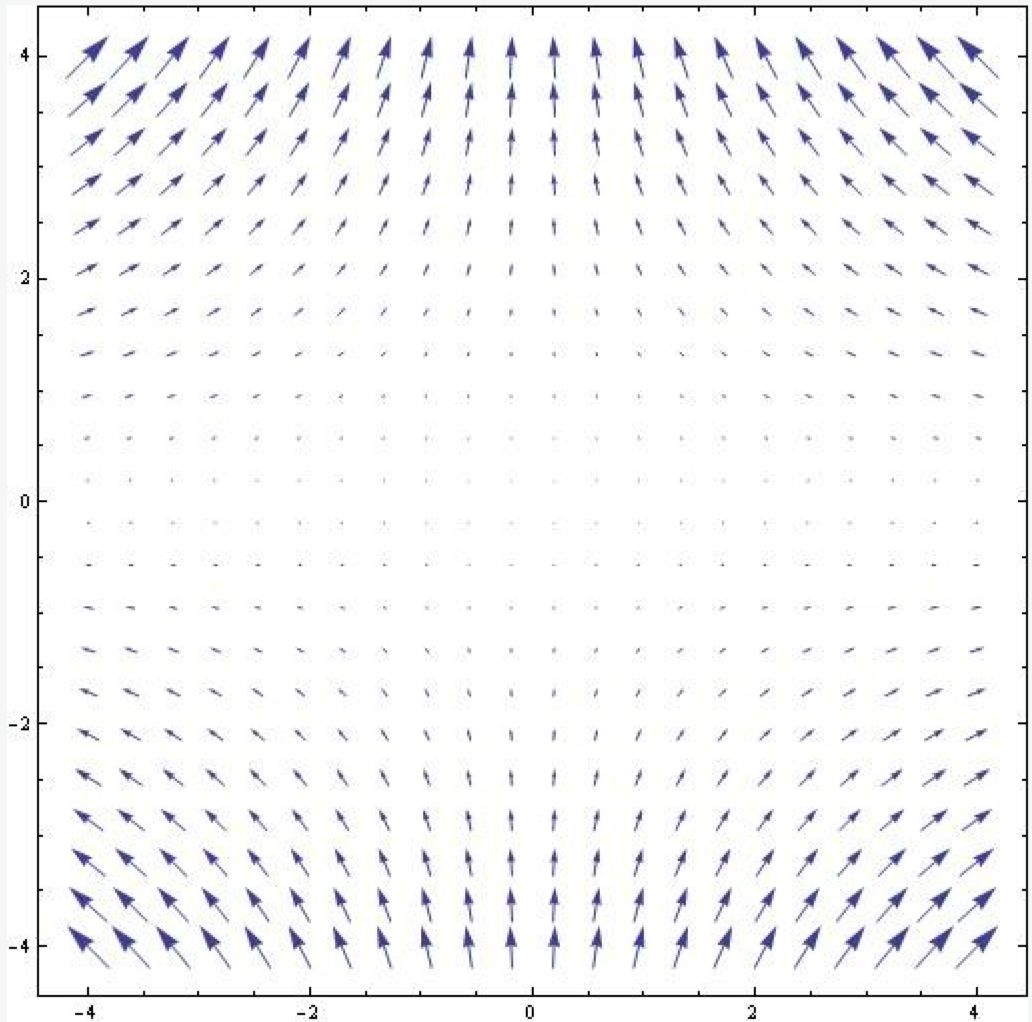


Abb. L12-1: Das Vektorfeld der Funktion ***F*** *(x, y)*

$$\vec{F} = -x\,y\,\vec{i} + y^2 \cdot \vec{j}\,, \qquad div\,\vec{F} = y$$

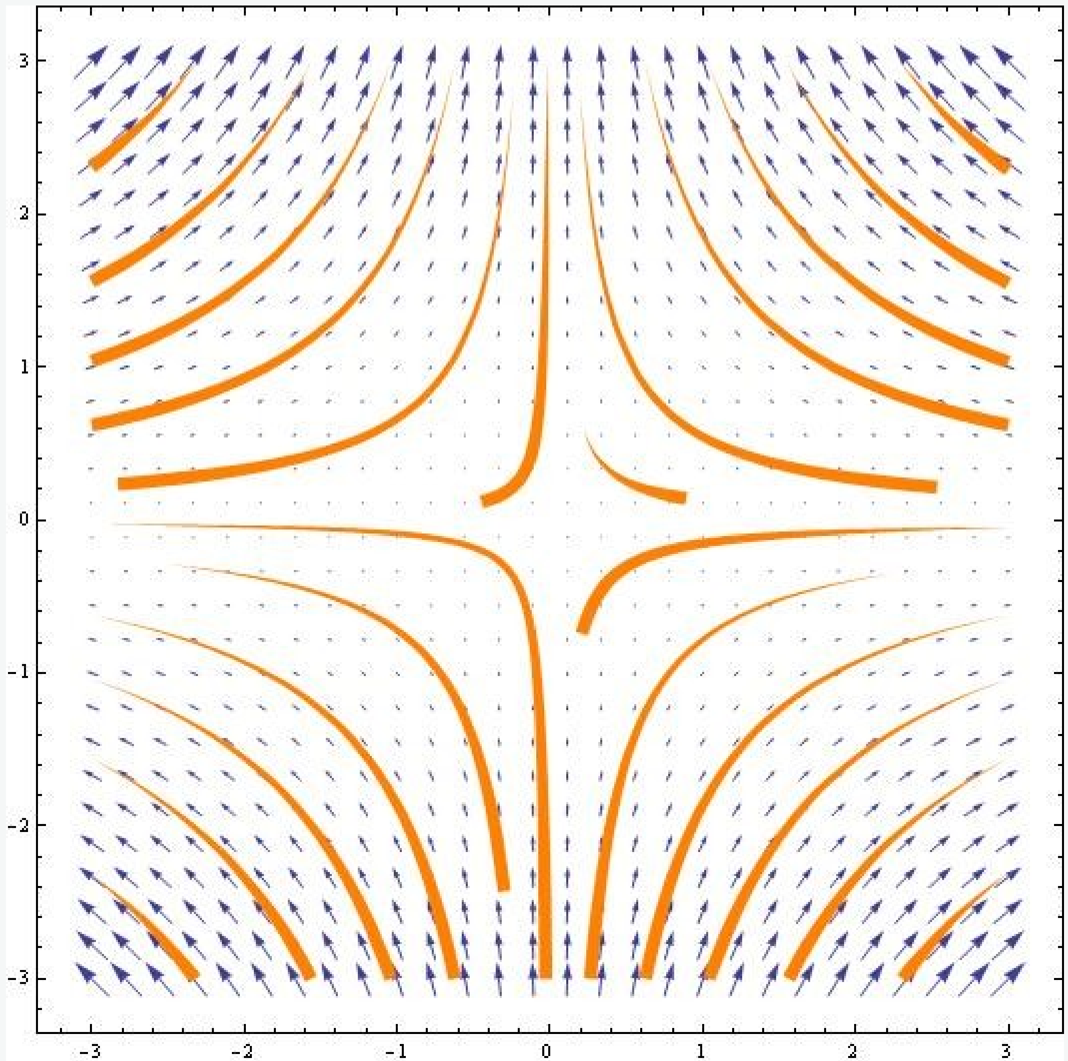


*Abb. L12-2: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

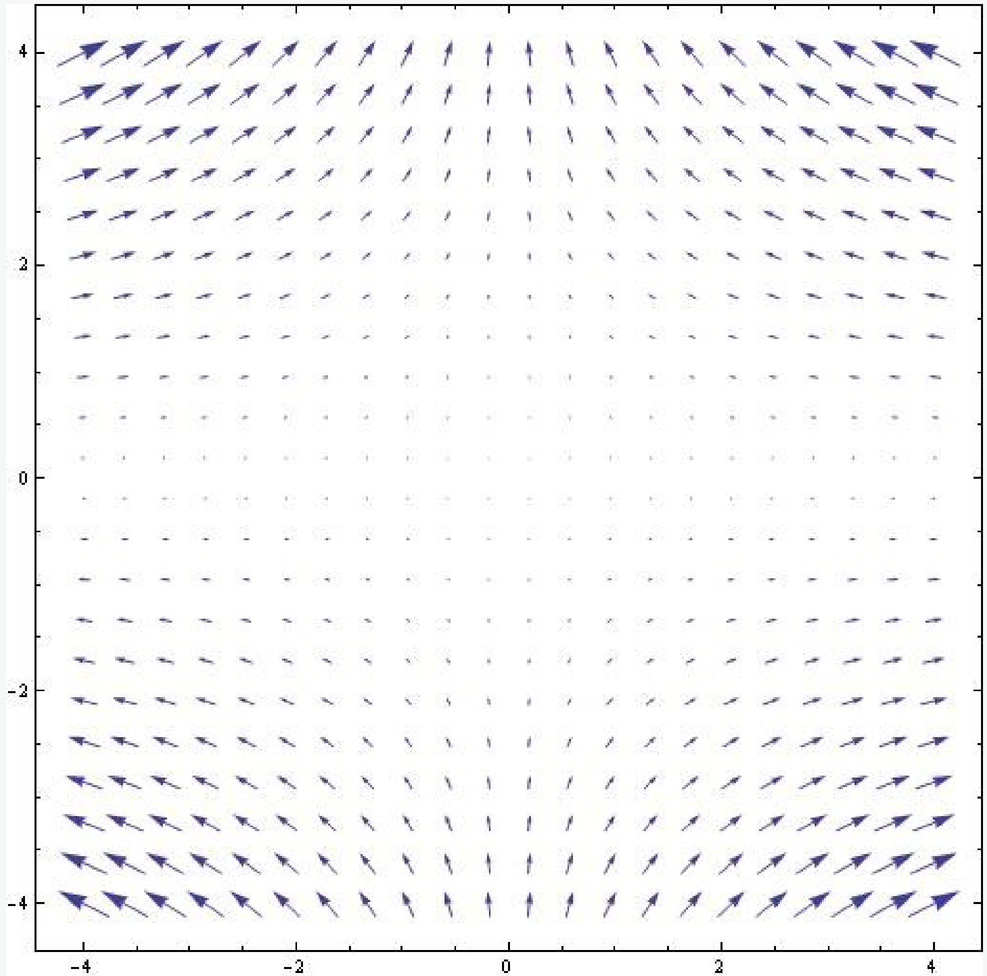


Abb. L13-1: Das Vektorfeld der Funktion ***F*** *(x, y)*

$$\vec{F} = -2\,x\,y\;\vec{i} + y^2 \cdot \vec{j}\,, \qquad div\,\vec{F} = 0$$

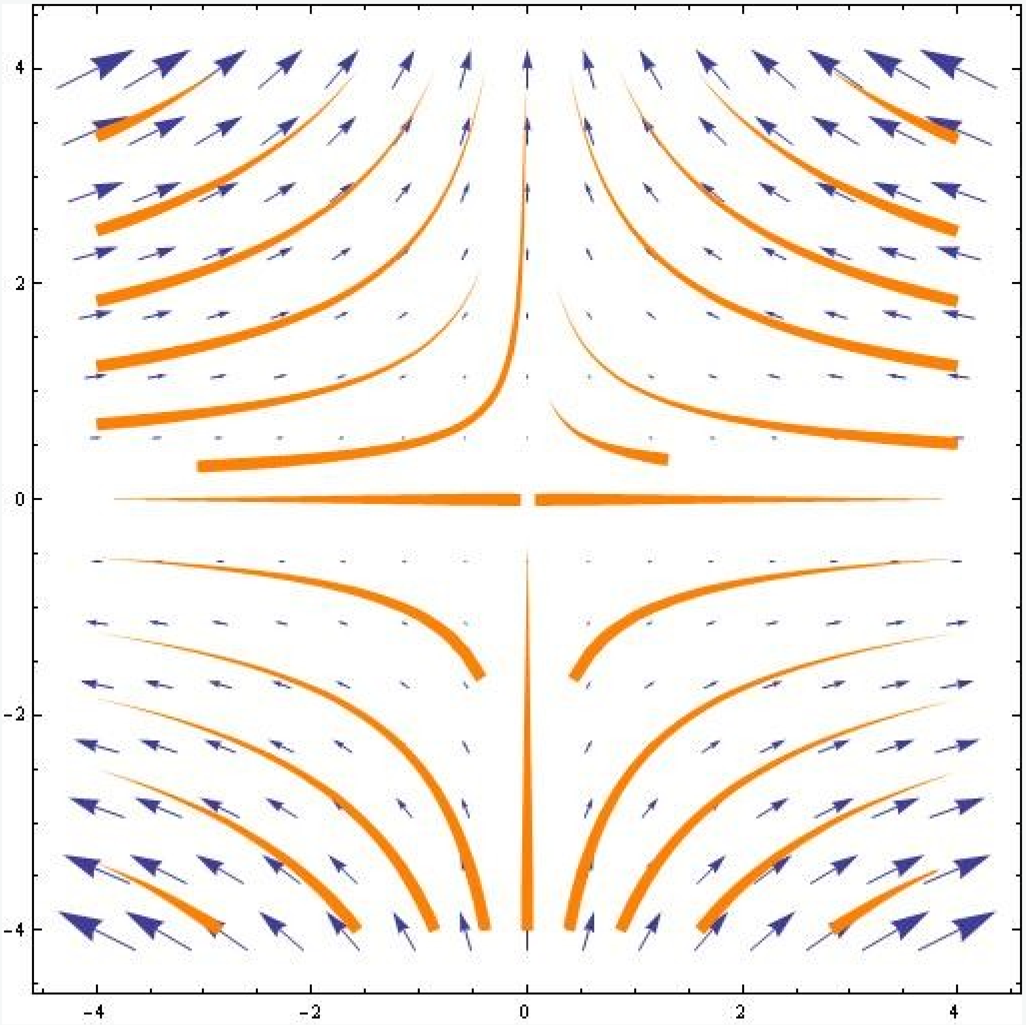


*Abb. L13-2: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

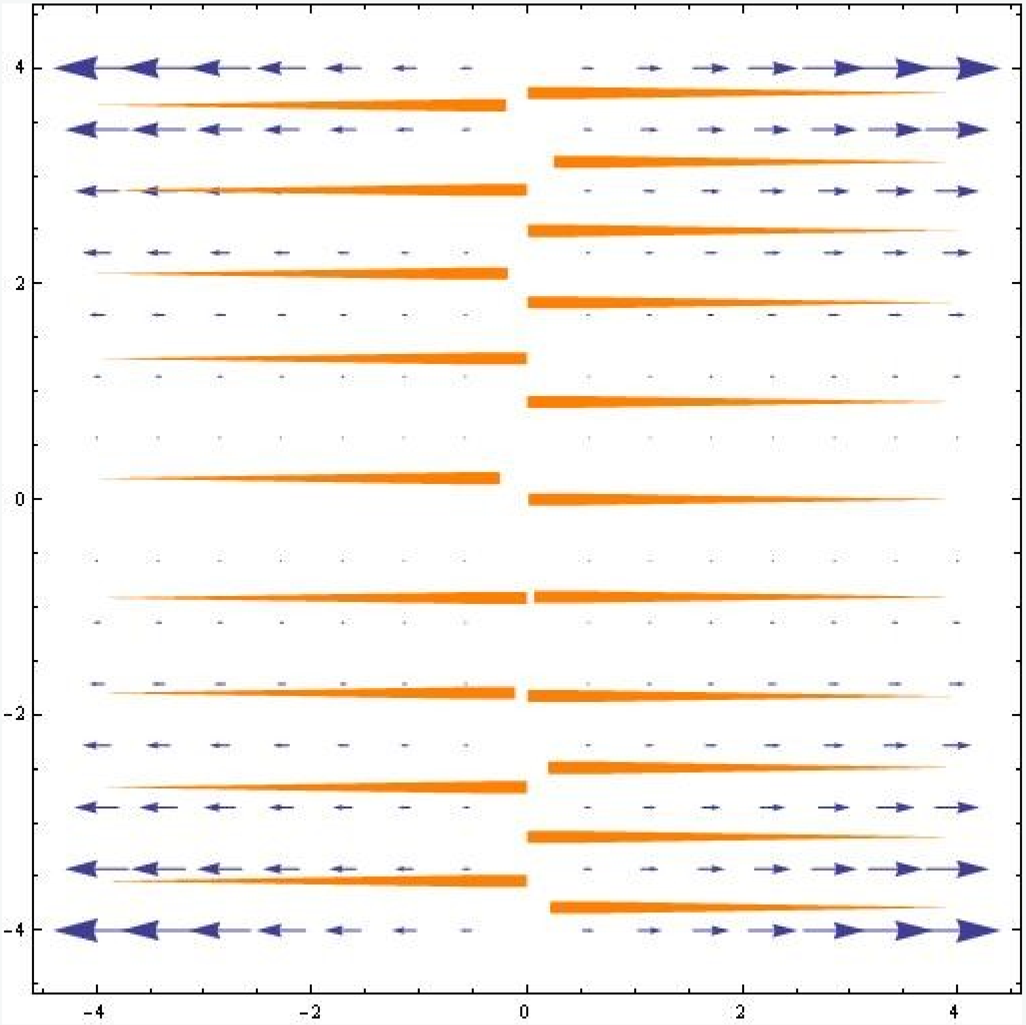


*Abb. L14: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

$$\vec{F} = x\, y^2\, \vec{i}\,, \qquad div\, \vec{F} = y^2$$

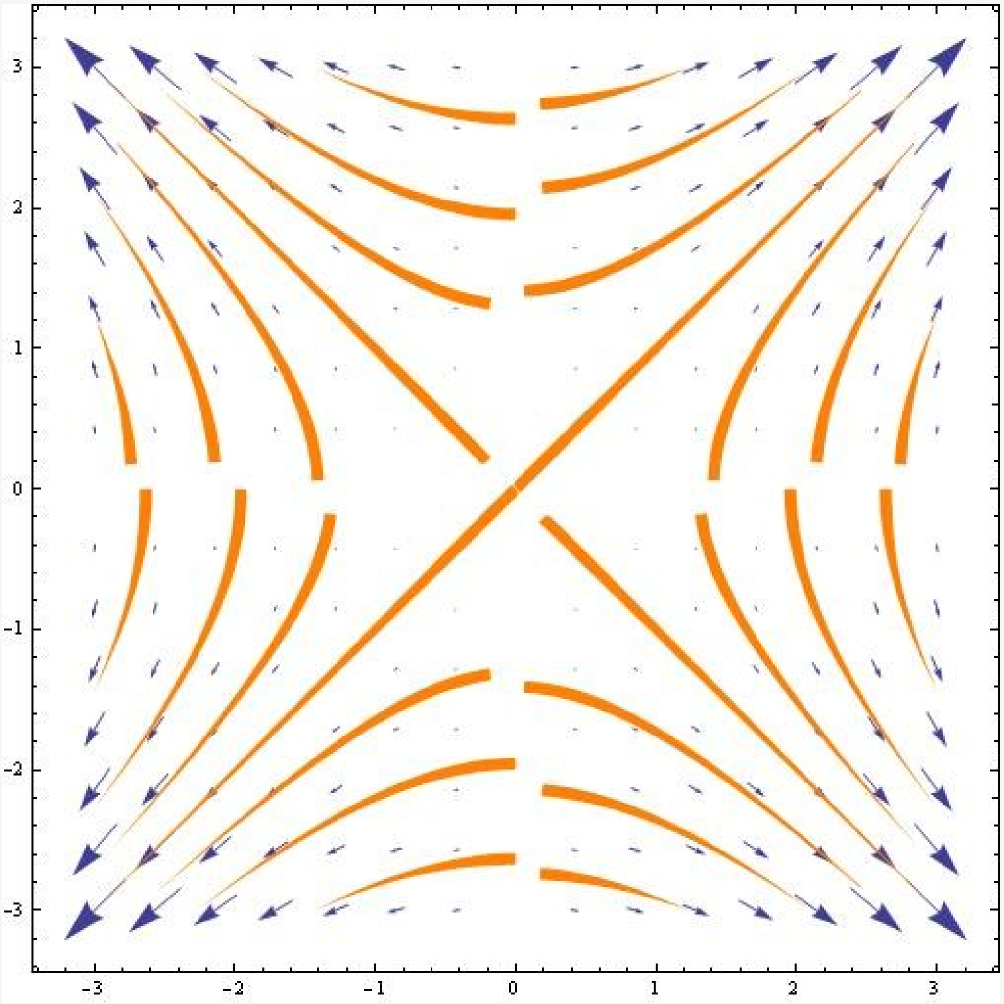


*Abb. L15-1: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*

$$\vec{F} = x\,y^2\,\vec{i} + x^2\,y\,\vec{j}\,, \qquad div\,\vec{F} = x^2 + y^2$$

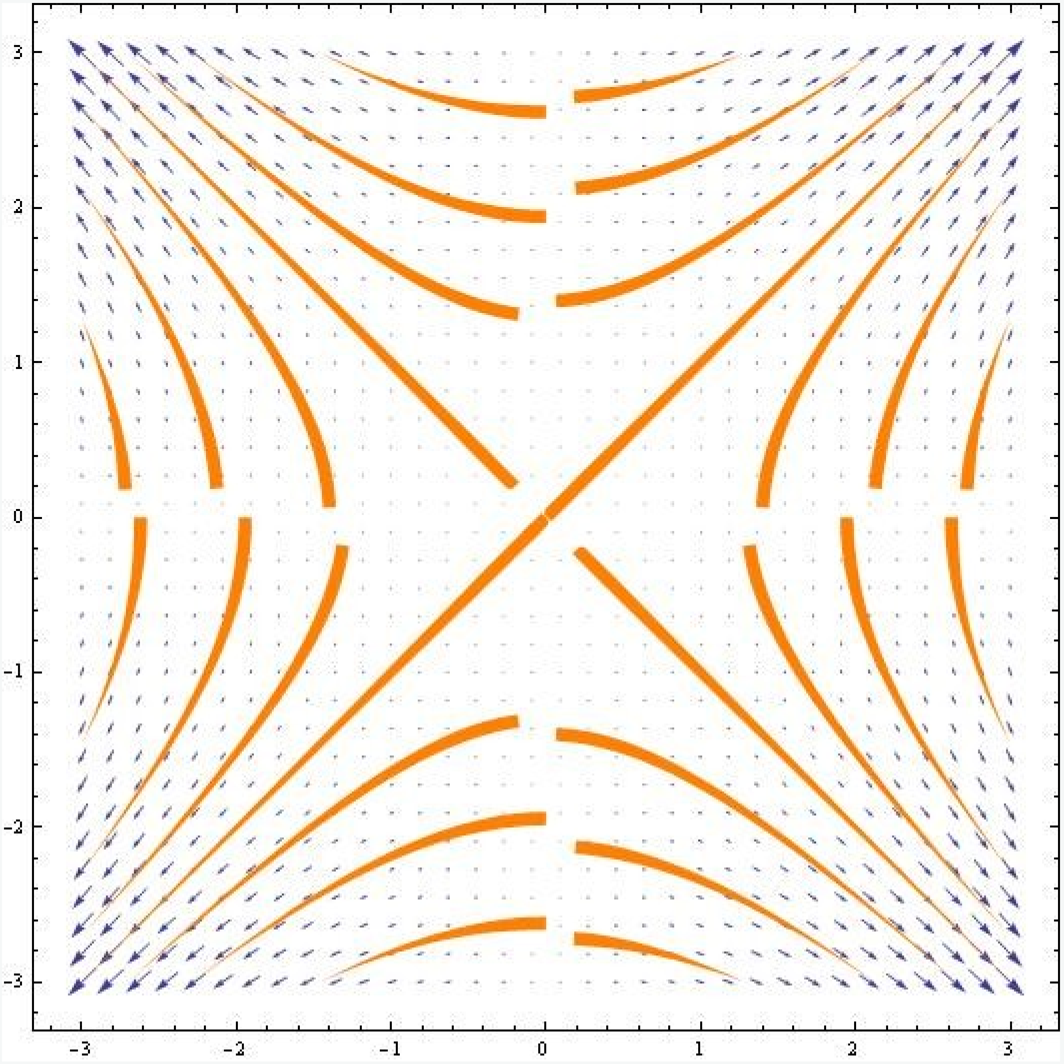
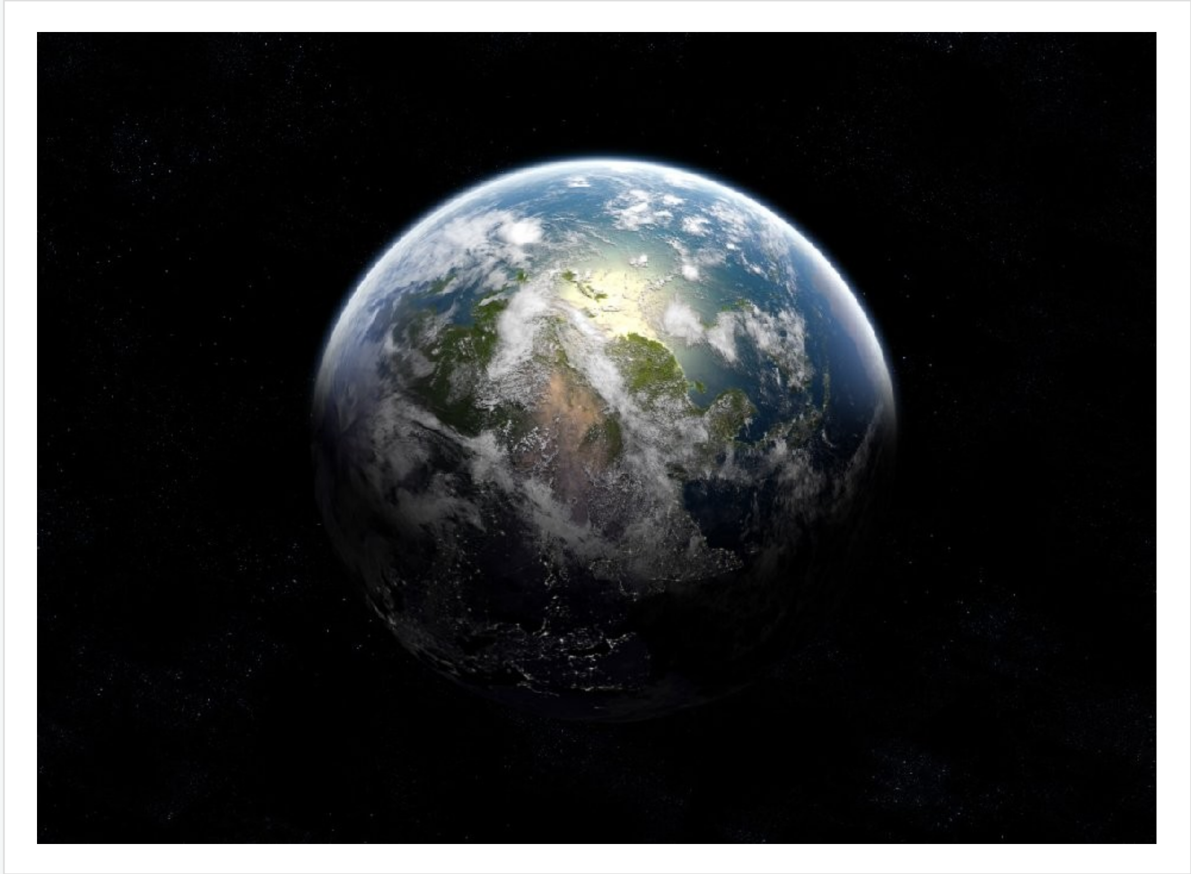


*Abb. L15-2: Das Vektorfeld und Feldlinien der Funktion **F** (x, y)*



http://fc51.deviantart.com/fs26/f/2008/143/0/c/Planet_Earth_by_sanmonku.jpg

Aufgabe 16: Berechnen Sie die Divergenz des Gravitationsfeldes der Erde.

$$\vec{F} = -\gamma \, \frac{m\,M}{r^2} \, \frac{\vec{r}}{r} \quad \text{– Gravitationskraft der Erde}$$

M – Erdmasse, r – Abstand einer Masse m vom Erdmittelpunkt

$$\vec{F} = -\gamma \, \frac{m\,M}{r^2} \, \frac{\vec{r}}{r} = C \, \frac{\vec{r}}{r^3}\,, \qquad C \equiv -\gamma \; m\,M$$

Divergenz: Lösung 16

$$\vec{F} = \frac{C}{r^3}\,\vec{r} = \frac{C}{r^3}\left(x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z\right), \qquad r \neq 0$$

$$div\,\vec{F} = C\left\{\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)\right\}$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{1}{r^6}\left(r^3 - x\cdot 3\,r^2\cdot\frac{\partial r}{\partial x}\right) = \frac{1}{r^3} - \frac{3\,x^2}{r^5}$$

$$\frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) = \frac{1}{r^3} - \frac{3\,y^2}{r^5}, \qquad \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right) = \frac{1}{r^3} - \frac{3\,z^2}{r^5}$$

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2 + y^2 + z^2} = \frac{1}{2}\left(x^2 + y^2 + z^2\right)^{-1/2} 2\,x = \frac{x}{r}$$

$$div\,\vec{F} = C\left\{\frac{3}{r^3} - \frac{3}{r^5}\left(x^2 + y^2 + z^2\right)\right\} = C\left\{\frac{3}{r^3} - \frac{3\,r^2}{r^5}\right\} = 0$$

Das Gravitationsfeld ist quellenfrei.