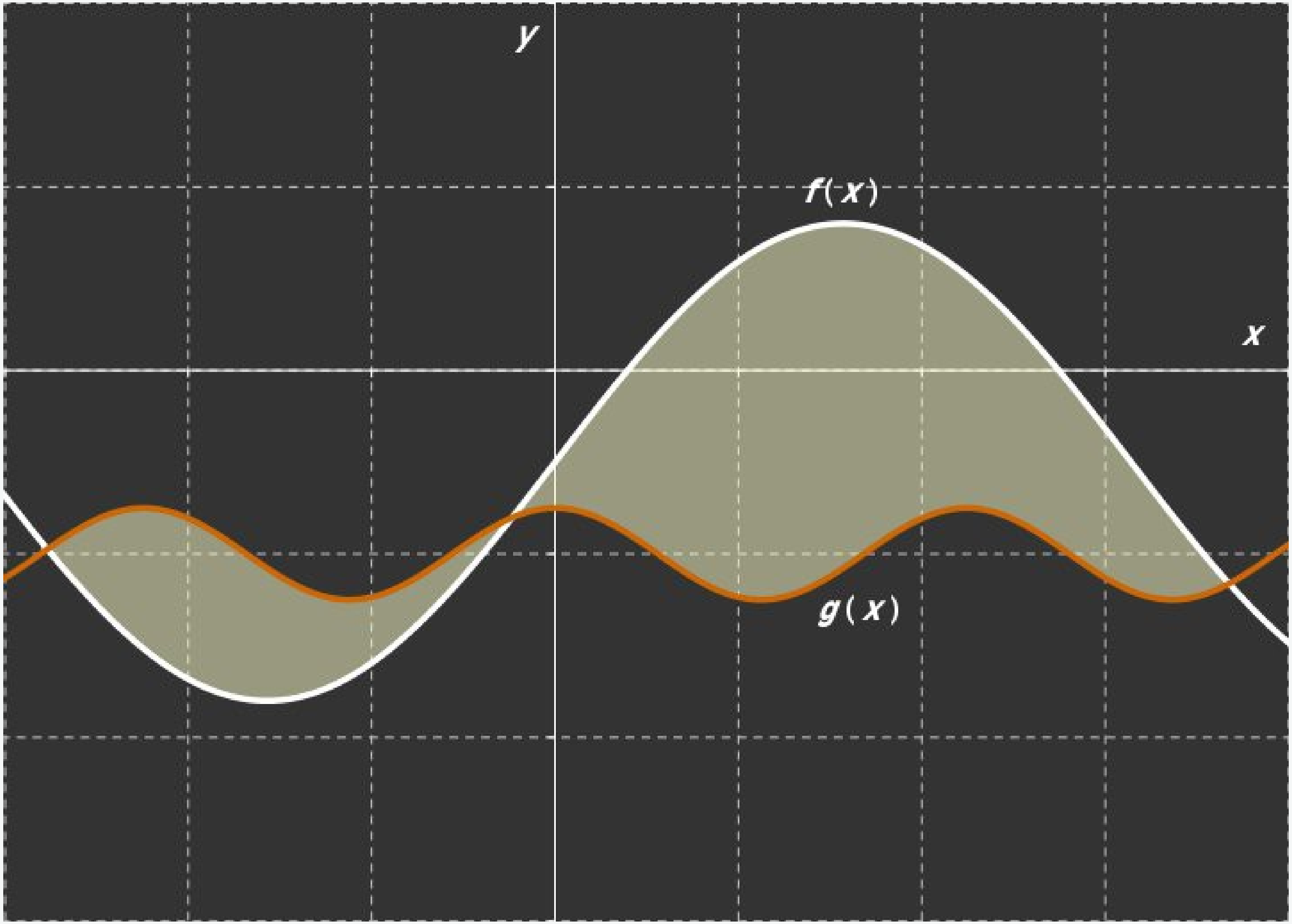

Flächen zwischen zwei Kurven: Aufgaben 20-24

Bestimmen Sie die Fläche, die im Intervall I von den Kurven mit den Gleichungen $y = f(x)$ und $y = g(x)$ begrenzt werden:

Aufgabe 20: $f(x) = 2\sin\left(\dfrac{x}{2}\right) + 2, \quad g(x) = \cos x, \quad I = \left[-\dfrac{\pi}{2}, \dfrac{5\pi}{2} \right]$

Aufgabe 21: $f(x) = 4\sin^2\left(\dfrac{x}{4}\right), \quad g(x) = \dfrac{1}{20}\, x\,(x - 4\pi), \quad I = [0, 4\pi]$

Aufgabe 22: $f(x) = 4\sin^2\left(\dfrac{x}{4}\right), \quad g(x) = -1 + \dfrac{1}{5}\sin(4x)$

$I = [0, 4\pi]$

Aufgabe 23: $f(x) = 5\sin\left(\dfrac{x}{4}\right), \quad g(x) = 3\sin^3 x \cdot \cos x, \quad I = [0, 4\pi]$

Aufgabe 24: $f(x) = \sin x \cdot e^{\cos x} + 2, \quad g(x) = \dfrac{1}{5}\sin(4x) - 1$

$a)\ I = [0, \pi], \qquad b)\ I = [-2\pi, 2\pi]$

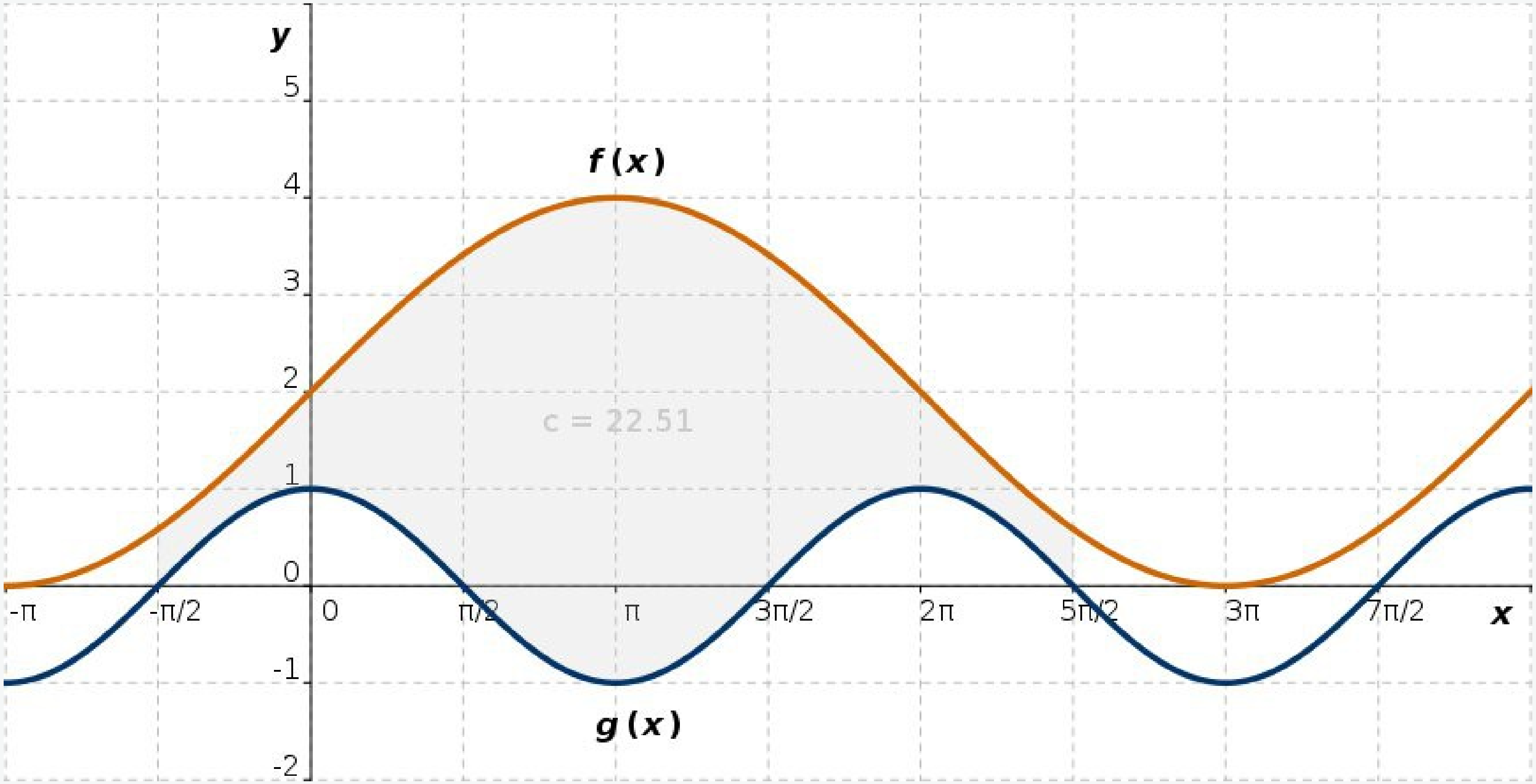

Abb. L20: Die Fläche zwischen den Kurven f (x) und g (x)

$$f(x) = 2\sin\left(\frac{x}{2}\right) + 2, \qquad g(x) = \cos x, \qquad I = \left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$$

$$f(x) = 2\sin\left(\frac{x}{2}\right) + 2, \qquad g(x) = \cos x, \qquad I = \left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$$

$$\int \sin(a\,x)\,dx = -\frac{\cos(a\,x)}{a}, \qquad I_1 = 2\int_{-\pi/2}^{5\pi/2} \sin\left(\frac{x}{2}\right) dx = 4\sqrt{2}$$

$$A = \int_{-\pi/2}^{5\pi/2} (f(x) - g(x))\,dx = \int_{-\pi/2}^{5\pi/2} \left(2\sin\left(\frac{x}{2}\right) + 2 - \cos x\right) dx =$$

$$= 4\sqrt{2} - 2 + 6\pi \text{ FE}$$

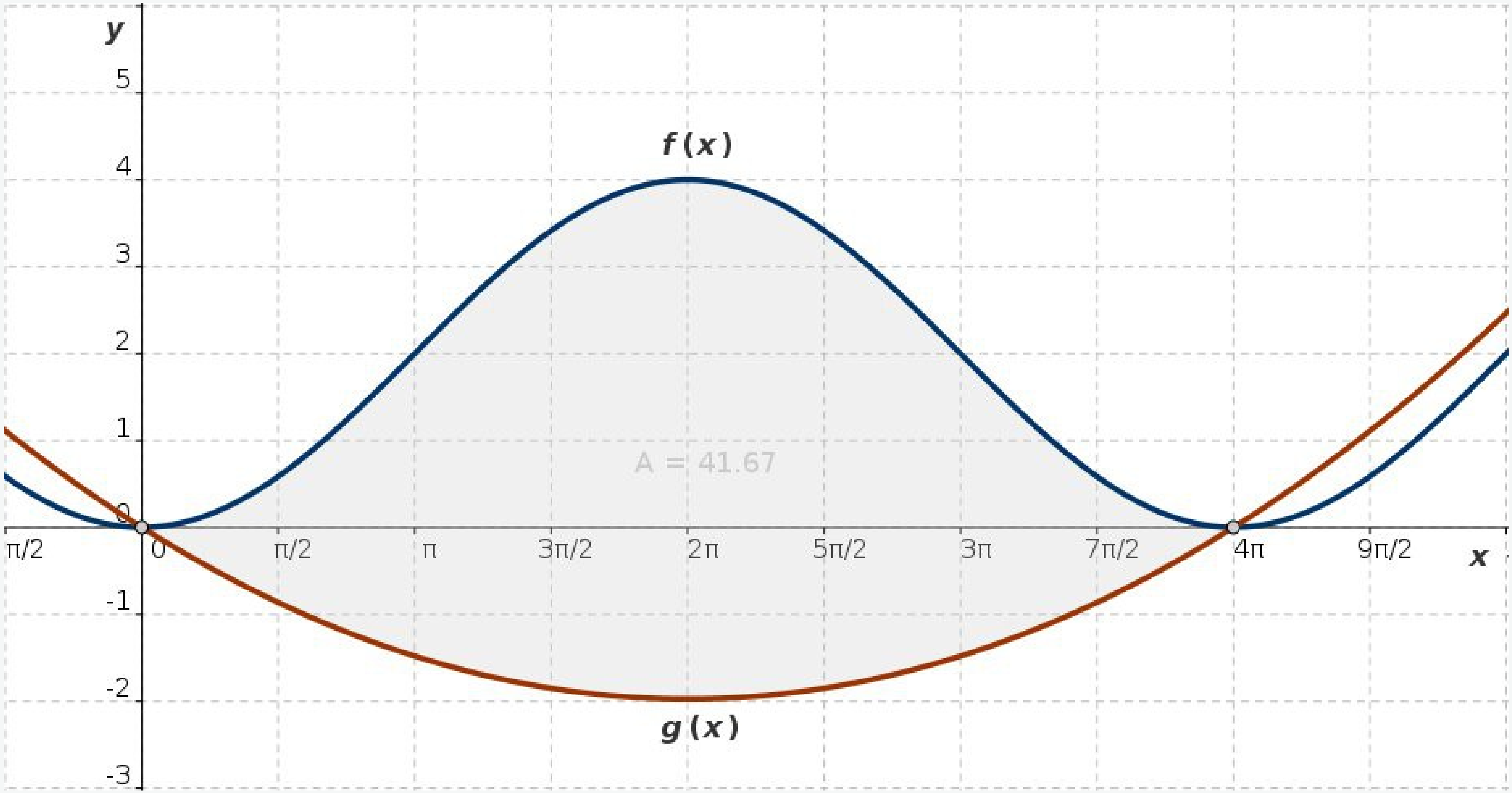

Abb. L21: Die Fläche zwischen den Kurven f (x) und g (x)

$$f(x) = 4\sin^2\left(\frac{x}{4}\right), \qquad g(x) = \frac{1}{20}\, x\,(x - 4\pi), \qquad I = [\,0,\ 4\pi\,]$$

$$f(x) = 4\sin^2\left(\frac{x}{4}\right), \qquad g(x) = \frac{1}{20}\, x\,(x - 4\pi), \qquad I = [\,0,\ 4\pi\,]$$

$$\int \sin^2(a\,x)\,dx = -\frac{1}{2\,a}\cos(a\,x)\,\sin(a\,x) + \frac{x}{2} = -\frac{\sin(2\,a\,x)}{4\,a} + \frac{x}{2}$$

$$I_1 = \int_0^{4\pi} \sin^2\left(\frac{x}{4}\right) dx = \left[-\sin\left(\frac{x}{2}\right) + \frac{x}{2}\right]_0^{4\pi} = 2\pi$$

$$I_2 = \int_0^{4\pi} x\,(x - 4\pi)\,dx =$$

$$A = \int_0^{4\pi} (f(x) - g(x))\,dx = \int_0^{4\pi} \left(4\sin^2\left(\frac{x}{4}\right) - \frac{x}{20}\,(x - 4\pi)\right) dx =$$

$$= 4\,I_1 - \frac{1}{20}\,I_2 = (?)\ \text{FE}$$

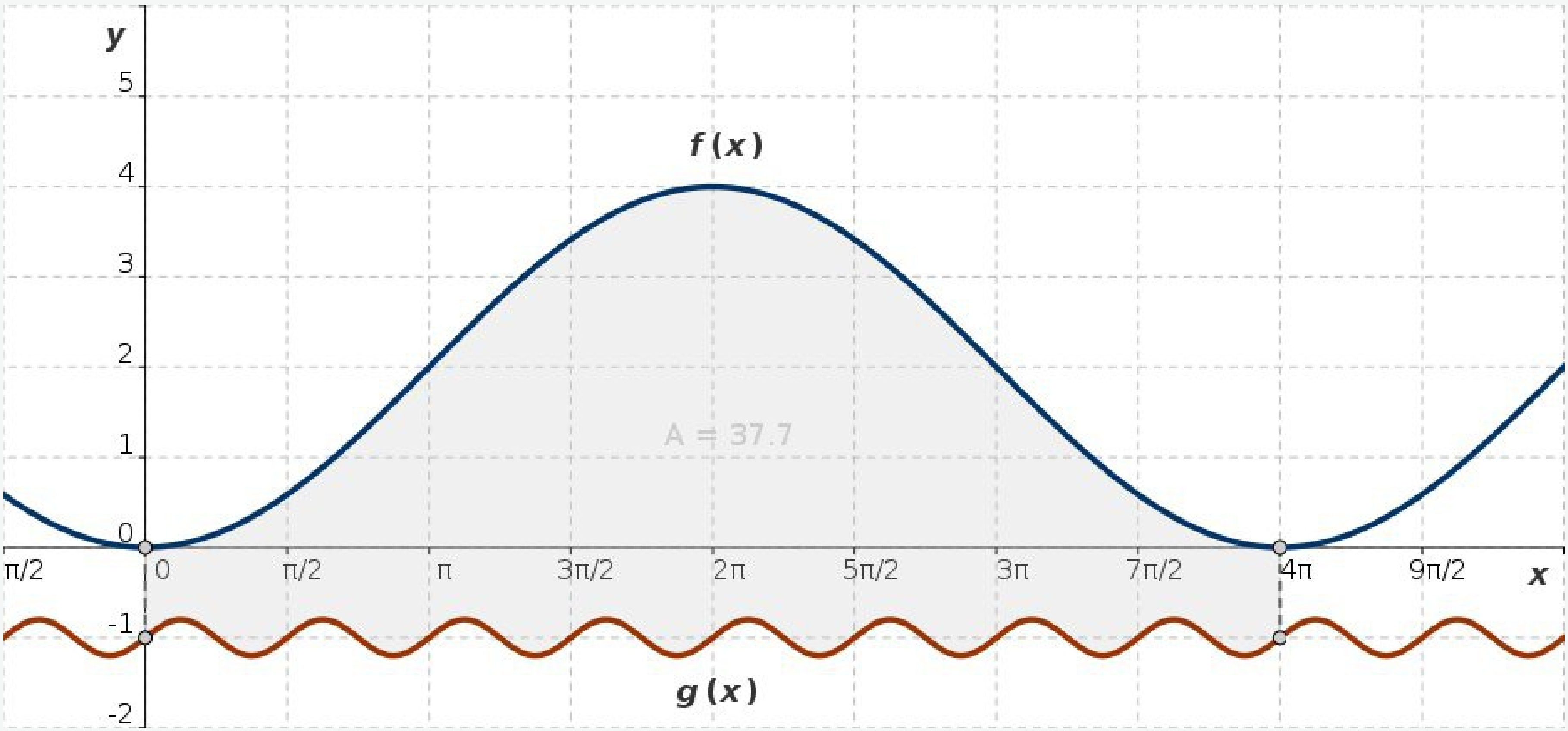

Abb. L22: Die Fläche zwischen den Kurven f (x) und g (x)

$$f(x) = 4\sin^2\left(\frac{x}{4}\right), \qquad g(x) = -1 + \frac{1}{5}\sin(4x), \qquad I = [\,0,\ 4\pi\,]$$

$$f(x) = 4\sin^2\left(\frac{x}{4}\right), \qquad g(x) = -1 + \frac{1}{5}\sin(4x), \qquad I = [\,0,\ 4\pi\,]$$

$$I_1 = \int_0^{4\pi} \sin^2\left(\frac{x}{4}\right) dx = \left[-\sin\left(\frac{x}{2}\right) + \frac{x}{2}\right]_0^{4\pi} = 2\pi$$

$$I_2 = \int_0^{4\pi} \sin(4x)\, dx = 0$$

$$A = \int_0^{4\pi} (f(x) - g(x))\, dx = \int_0^{4\pi} \left(4\sin^2\left(\frac{x}{4}\right) + 1 - \frac{1}{5}\sin(4x)\right) dx =$$

$$= 4 I_1 - \frac{1}{5} I_2 + 4\pi = 12\pi \text{ FE}$$

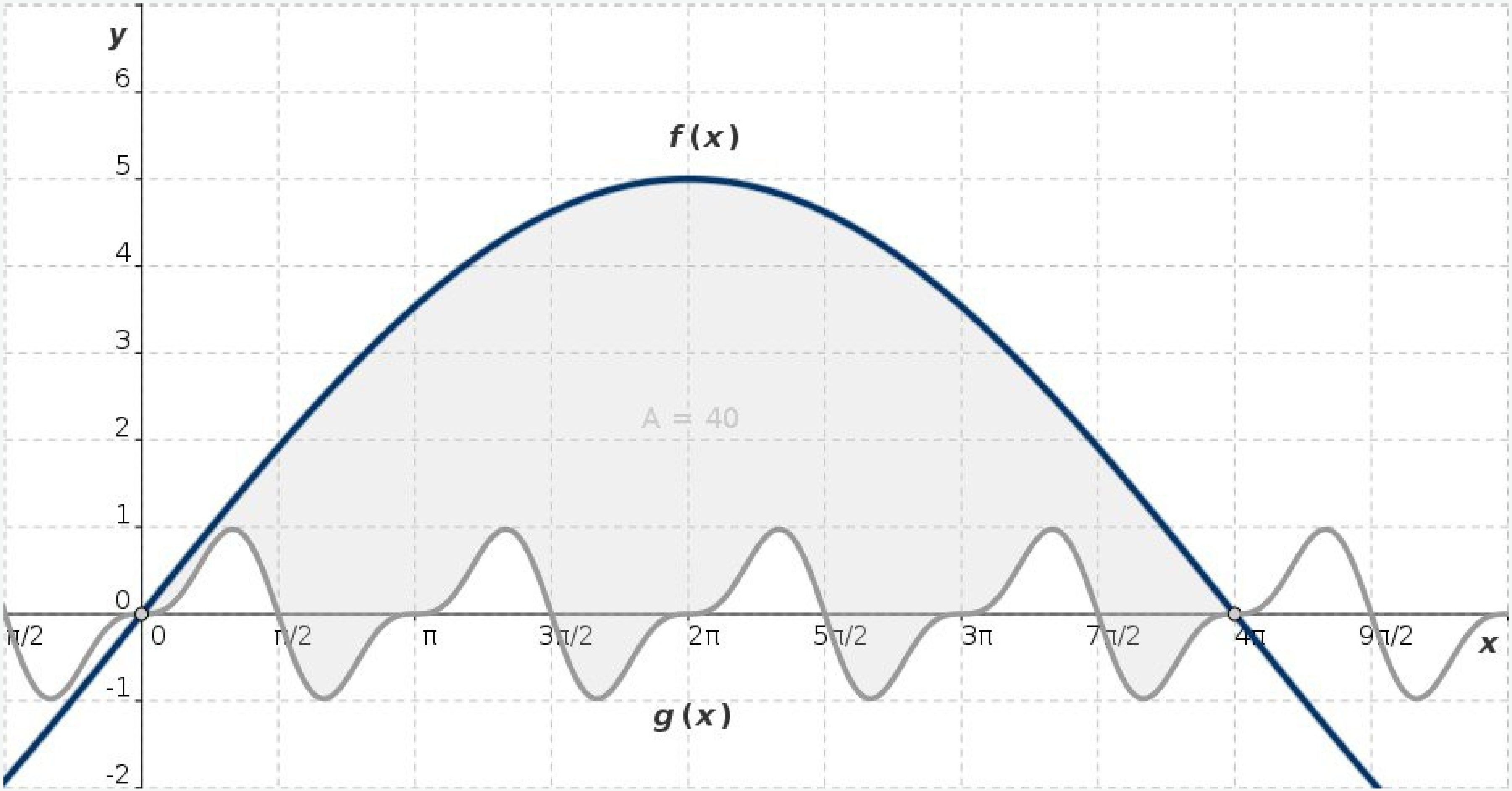

Abb. L23: Die Fläche zwischen den Kurven f (x) und g (x

$$f(x) = 5\sin\left(\frac{x}{4}\right), \qquad g(x) = 3\sin^3 x \cdot \cos x\,, \qquad I = [\,0,\ 4\pi\,]$$

$$I_1 = \int_0^{4\pi} \sin\left(\frac{x}{4}\right) dx = \left[-4\cos\left(\frac{x}{4}\right)\right]_0^{4\pi} = 8$$

$$\int \sin^3 x \cdot \cos x \, dx = \frac{\sin^4 x}{4}, \qquad I_2 = \int_0^{4\pi} \sin^3 x \cdot \cos x \, dx = 0$$

$$A = \int_0^{4\pi} (f(x) - g(x)) \, dx = \int_0^{4\pi} \left(5\sin\left(\frac{x}{4}\right) - 3\sin^3 x \cdot \cos x\right) dx =$$

$$= 5 I_1 - 3 I_2 = 40 \text{ FE}$$

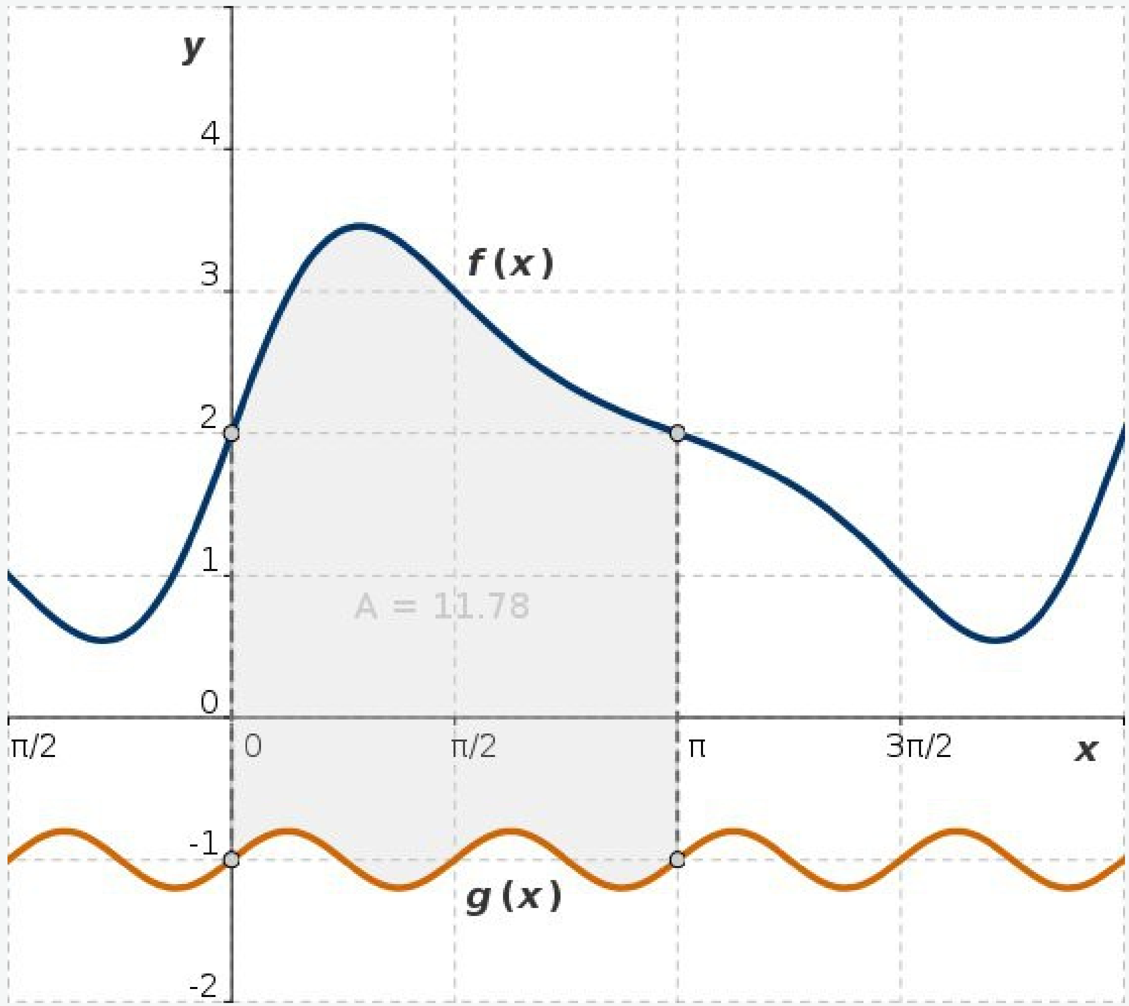

Abb. L24a: Die Fläche zwischen den Kurven f (x) und g (x)

$$f(x) = \sin x \cdot e^{\cos x} + 2, \qquad g(x) = \frac{1}{5} \sin(4x) - 1, \qquad I = [0, \pi]$$

$$f(x) = \sin x \cdot e^{\cos x} + 2, \qquad g(x) = \frac{1}{5} \sin(4x) - 1, \qquad I = [0, \pi]$$

$$\int \sin x \cdot e^{\cos x}\, dx = -e^{\cos x}, \qquad I_1 = \int_0^{\pi} \sin x \cdot e^{\cos x}\, dx = e - \frac{1}{e}$$

$$I_2 = \int_0^{\pi} \sin(4x)\, dx = 0$$

$$A = \int_0^{\pi} (f(x) - g(x))\, dx = \int_0^{\pi} \left(\sin x \; e^{\cos x} + 2 - \frac{1}{5} \sin(4x) + 1 \right) dx =$$

$$= I_1 - \frac{1}{5} I_2 + 3x \Big|_0^{\pi} = \left(e - \frac{1}{e} + 3\pi \right) \text{FE}$$

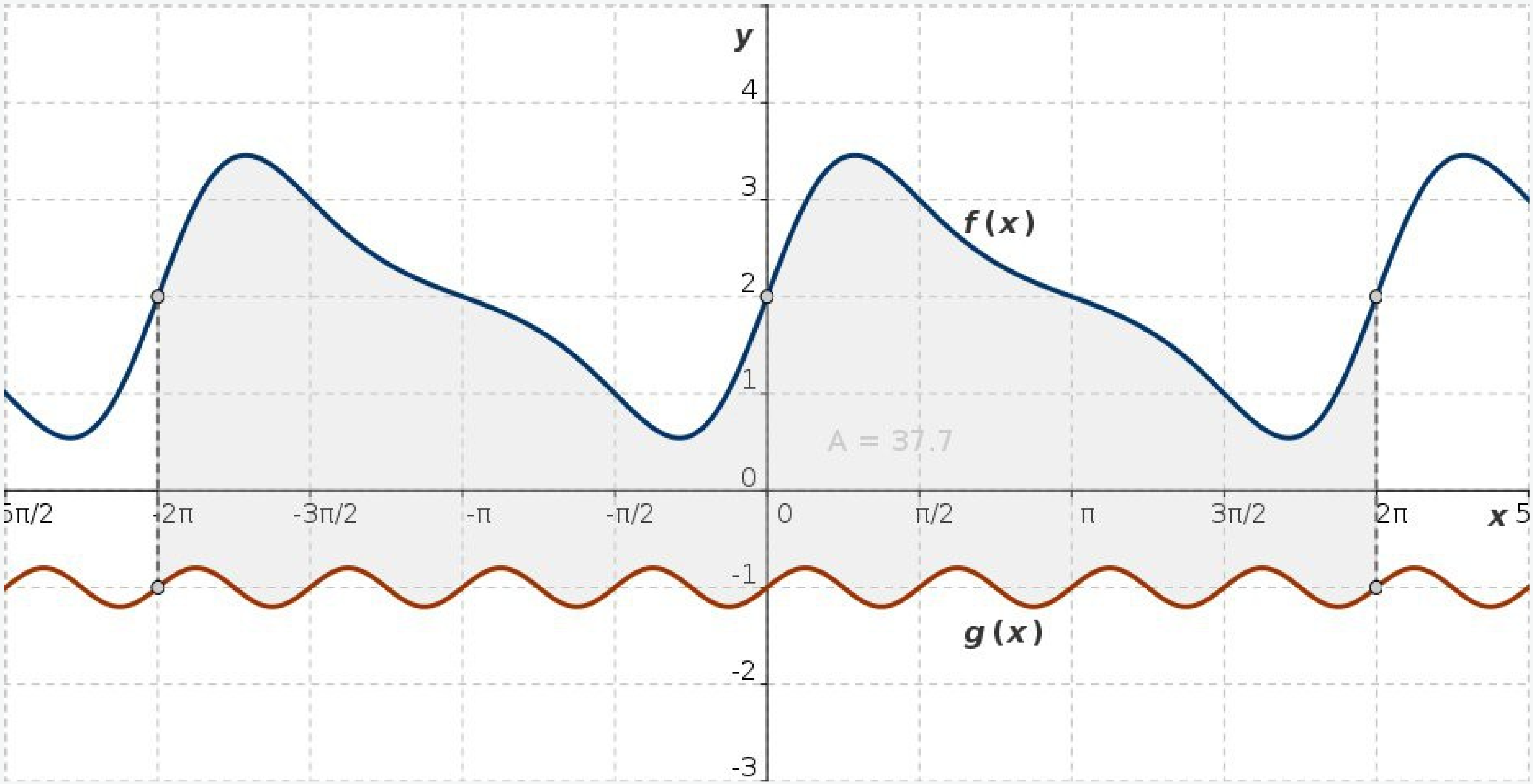

Abb. L24b: Die Fläche zwischen den Kurven f (x) und g (x)

$$f(x) = \sin x \cdot e^{\cos x} + 2, \qquad g(x) = \frac{1}{5}\sin(4x) - 1, \qquad I = [-2\pi, \ 2\pi]$$

$$f(x) = \sin x \cdot e^{\cos x} + 2, \qquad g(x) = \frac{1}{5} \sin(4x) - 1, \qquad I = [-2\pi, 2\pi]$$

$$\int \sin x \cdot e^{\cos x} \, dx = -e^{\cos x}, \qquad I_1 = \int_{-2\pi}^{2\pi} \sin x \cdot e^{\cos x} \, dx = 0$$

$$I_2 = \int_0^{4\pi} \sin(4x) \, dx = 0$$

$$A = \int_{-2\pi}^{2\pi} (f(x) - g(x)) \, dx = \int_{-2\pi}^{2\pi} \left(\sin x \; e^{\cos x} + 2 - \frac{1}{5} \sin(4x) + 1 \right) dx =$$

$$= I_1 - \frac{1}{5} I_2 + 3x \,\Big|_{-2\pi}^{2\pi} = 12\pi \ \text{FE}$$