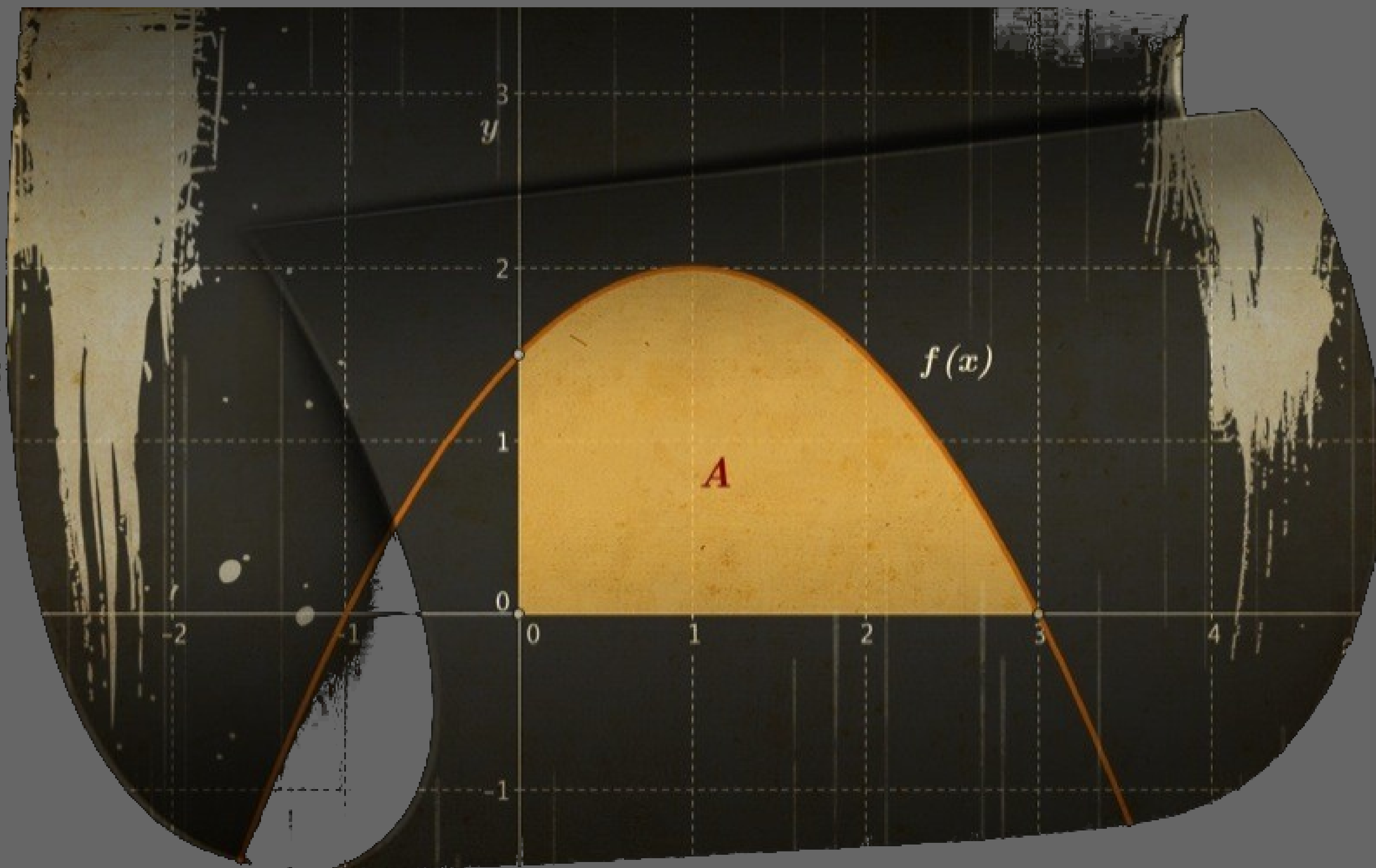


Bestimmtes Integral und Flächeninhalt

Teil 1

Bestimmen Sie den Flächeninhalt A, der von der Funktion $f(x)$ und der x-Achse und den Parallelen $x = a$ und $x = b$ begrenzt wird

Aufgabe 1: $f(x) = 0.5\, x^2 + 1$

$a)\ a = -1, \quad b = 2, \qquad b)\ a = -2, \quad b = 2$

Aufgabe 2: $f(x) = 0.5\, x^2 - 2, \qquad a = 2, \quad b = 4$

Aufgabe 3: $f(x) = -\dfrac{(x-1)^2}{2} + 2, \qquad a = 0, \quad b = 3$

Aufgabe 4: $f(x) = \sqrt{x}\,, \qquad a = 1, \quad b = 4$

Aufgabe 5: $f(x) = 2^{|x|}, \qquad a = -1, \quad b = 1$

Aufgabe 6: $f(x) = \sqrt{|x|} + 1, \qquad a = -1, \quad b = 4$

Aufgabe 7: $f(x) = \cos x + 2, \quad a = -\pi, \quad b = \pi$

Aufgabe 8: $f(x) = 2 - \cos x, \quad a = -\pi/2, \quad b = 3\pi/2$

Aufgabe 9: $f(x) = \sin x + 1, \quad a = -\pi, \quad b = \pi$

Aufgabe 10: $f(x) = |x|, \quad a = -2, \quad b = 2$

Aufgabe 11: $f(x) = \frac{|x|}{2} + 1, \quad a = -1, \quad b = 3$

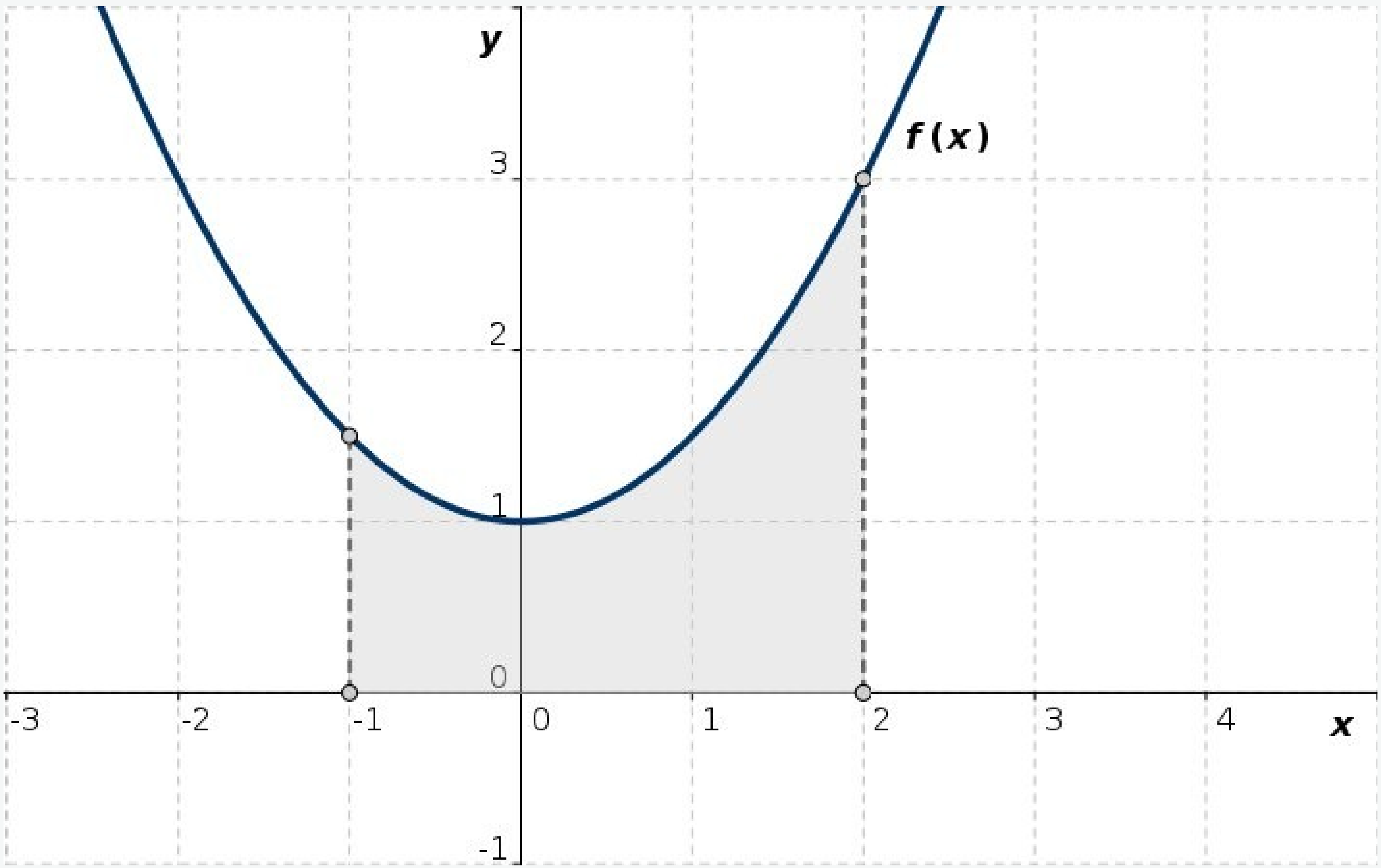


Abb. 1-1a: Zur Berechnung der Fläche unter der Kurve f (x) = 0.5 x² + 1 im Intervall [-1, 2]

$$A = \int_{-1}^{2} \left(\frac{1}{2} \, x^2 + 1 \right) dx = \left[\frac{1}{2} \, \frac{x^3}{3} + x \right]_{-1}^{2} = \frac{9}{2} = 4.5 \text{ FE}$$

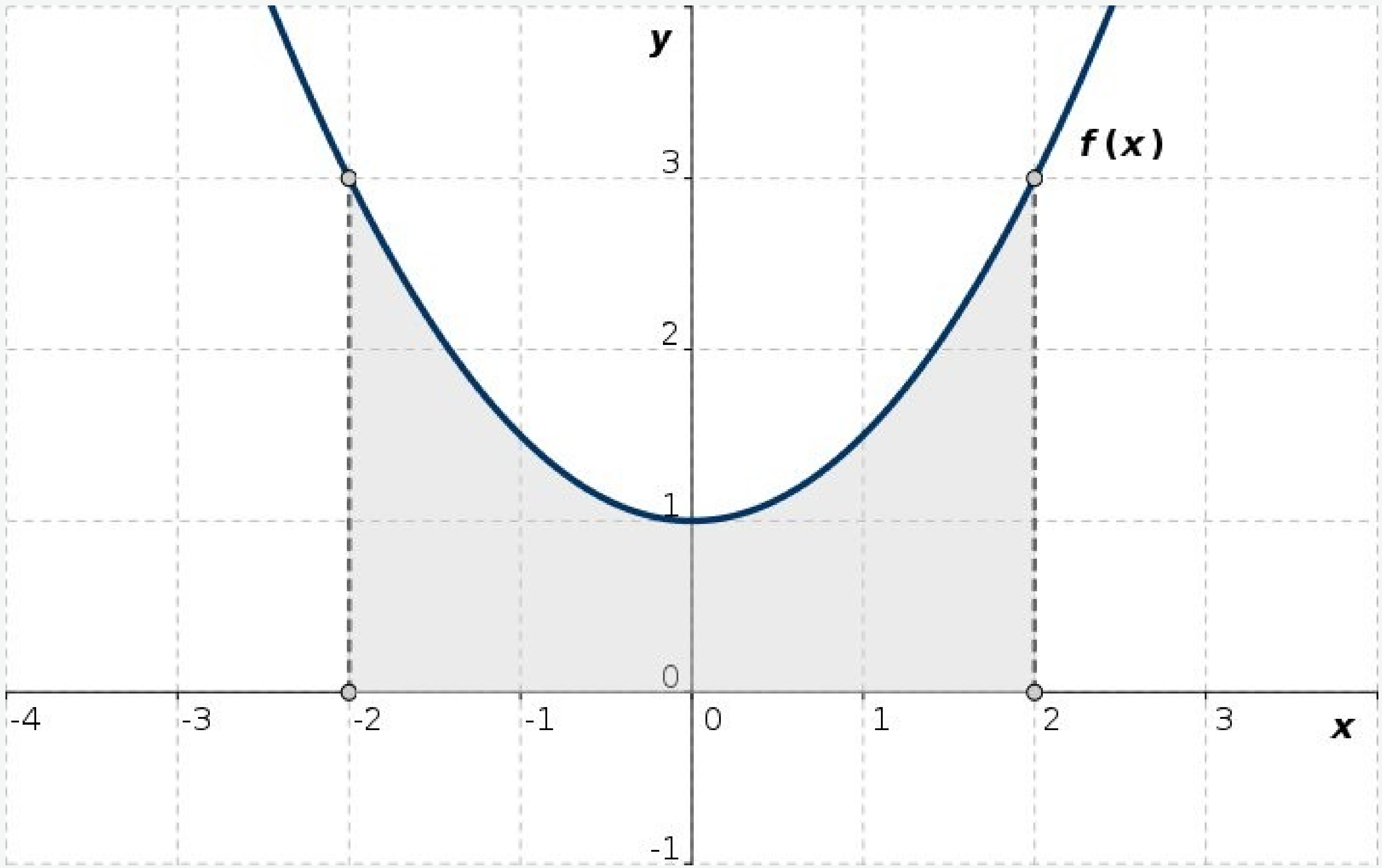


Abb. 1-1b: Zur Berechnung der Fläche unter der Kurve f (x) = 0.5 x² + 1 im Intervall [-2, 2]

$$A = \int_{-2}^{2} \left(\frac{1}{2}\, x^2 + 1 \right) dx = 2 \int_{0}^{2} \left(\frac{1}{2}\, x^2 + 1 \right) dx = 2 \left[\frac{1}{2}\, \frac{x^3}{3} + x \right]_0^2 = \frac{20}{3} \simeq 6.67 \ \text{FE}$$

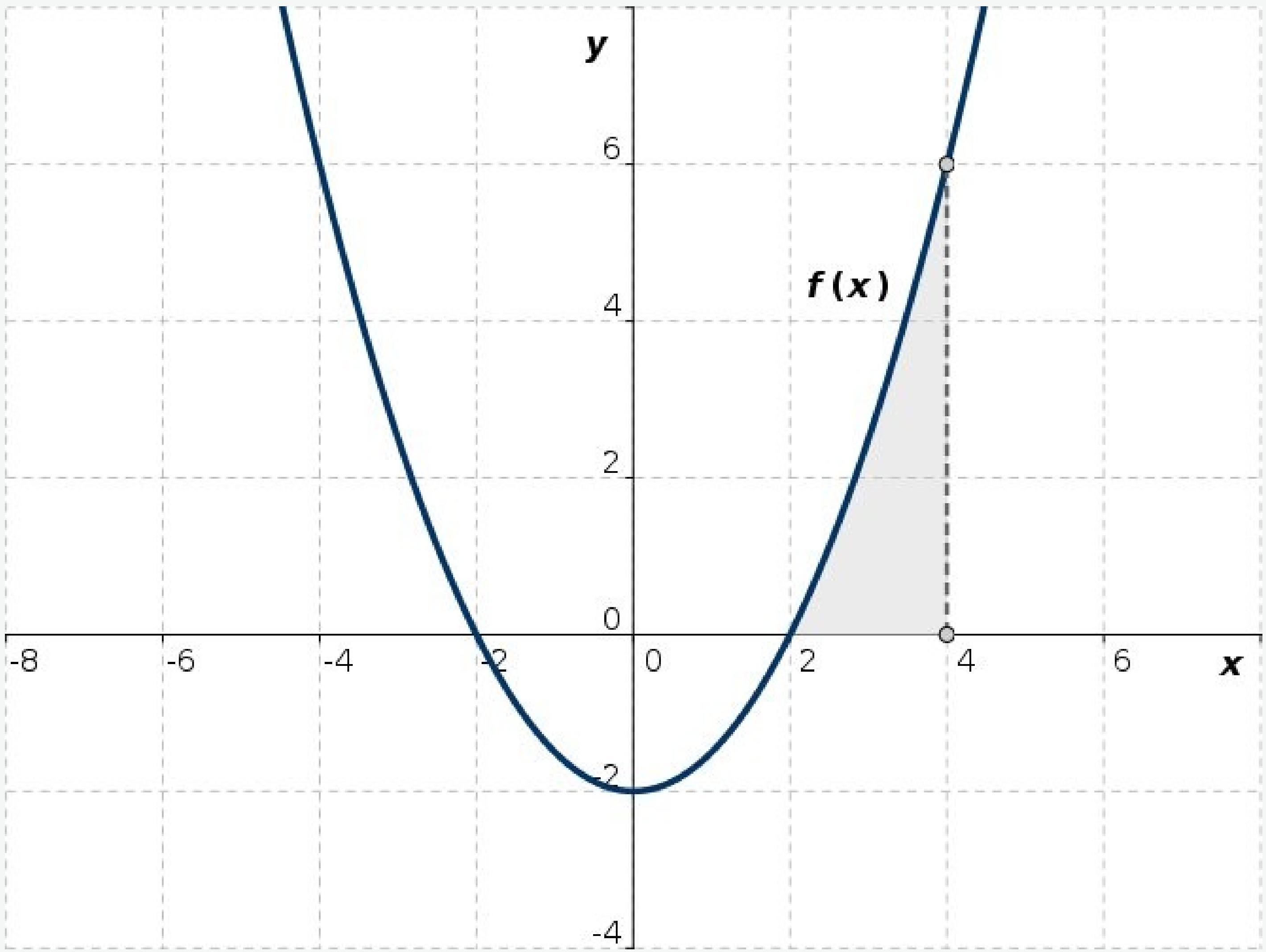


Abb. 1-2: Zur Berechnung der Fläche unter der Kurve f (x) = 0.5 x² - 2 im Intervall [2, 4]

$$A = \int_2^4 \left(\frac{1}{2}\, x^2 - 2 \right) dx = \left[\frac{1}{2}\, \frac{x^3}{3} - 2\, x \right]_2^4 = \frac{16}{3} \simeq 5.3 \;\; \text{FE}$$

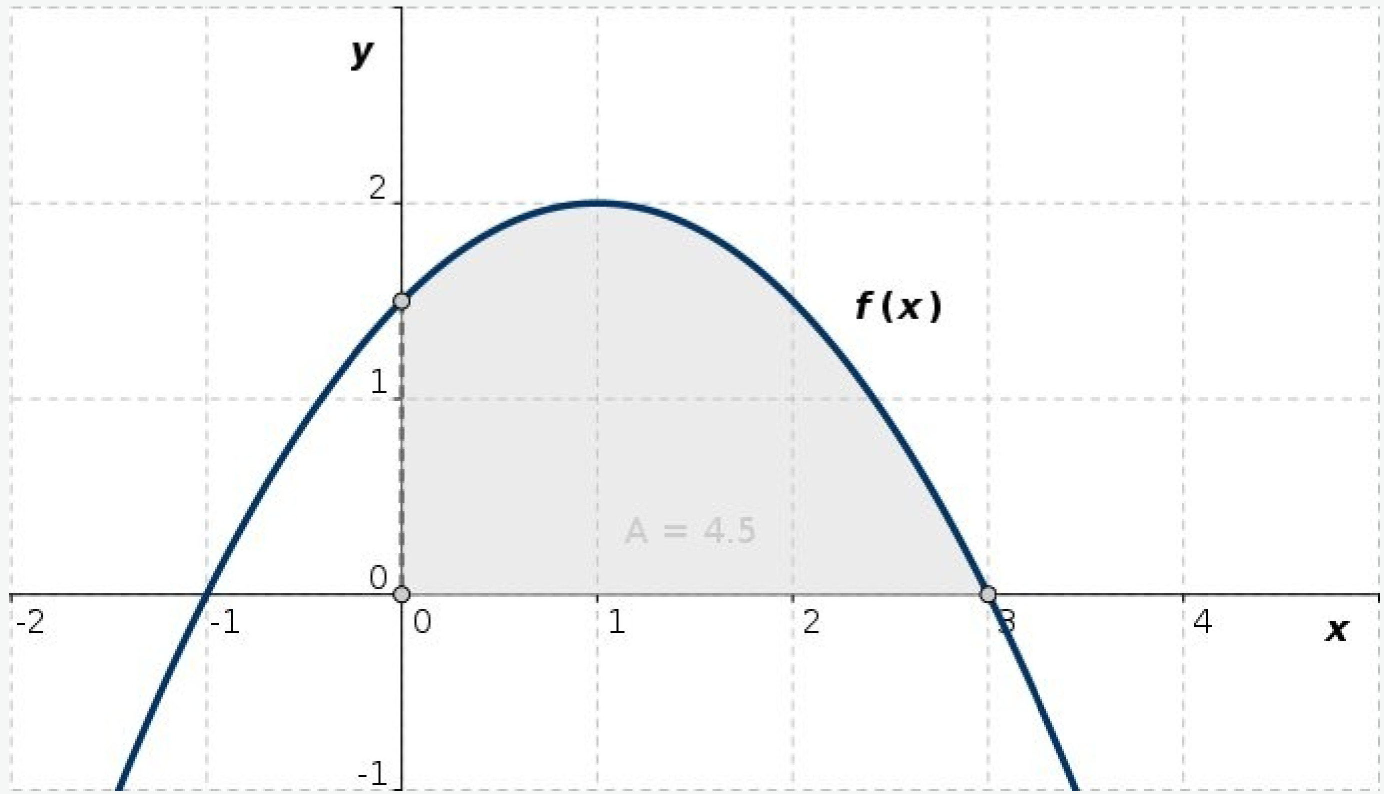


Abb. 1-3: Zur Berechnung der Fläche unter der Kurve f (x) = - (x – 1)²/2 + 2 im Intervall [0, 3]

$$A = \int_0^3 \left(- \frac{(x-1)^2}{2} + 2 \right) dx = \left[- \frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{2}\, x \right]_0^3 = \frac{9}{2} = 4.5 \ \text{FE}$$

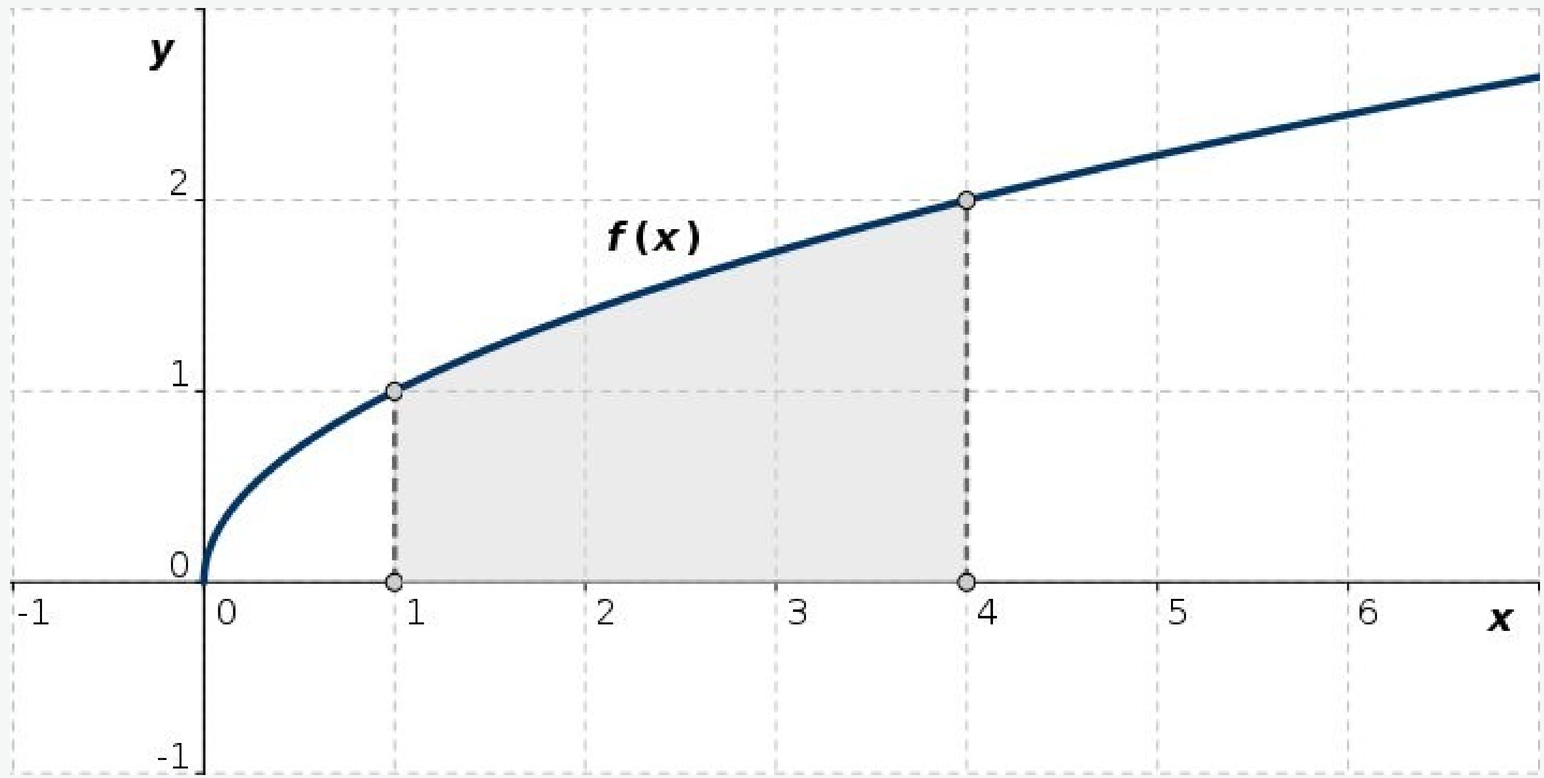


Abb. 1-4: Zur Berechnung der Fläche unter der Kurve $f(x) = \sqrt{x}$ im Intervall [1, 4]

$$A = \int_1^4 \sqrt{x}\; dx = \int_1^4 x^{\frac{1}{2}}\; dx = \frac{2}{3}\left(x\sqrt{x}\right)\Big|_1^4 = \frac{14}{3} \simeq 4.67 \text{ FE}$$

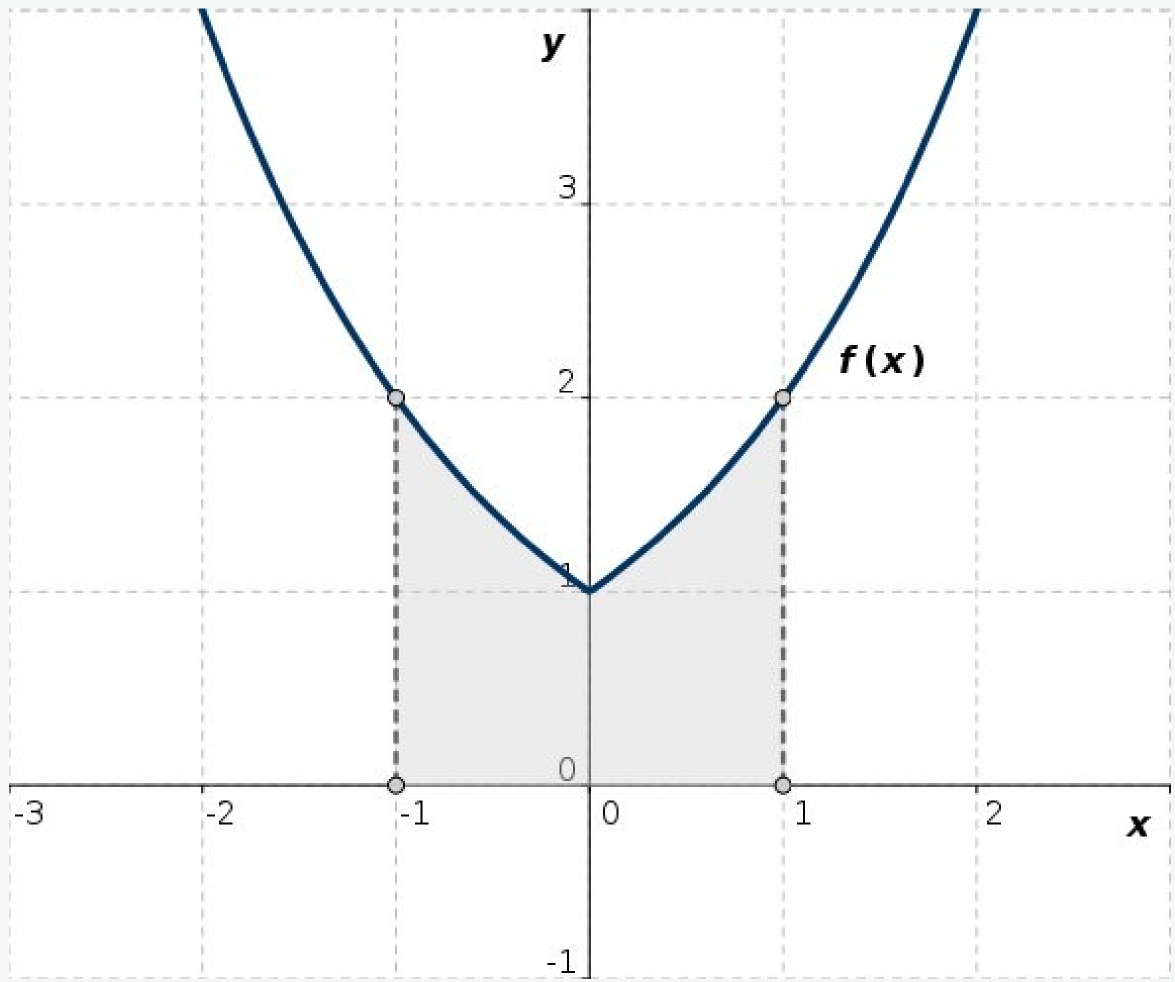


Abb. 1-5: Zur Berechnung der Fläche unter der Kurve y = f (x) im Intervall [-1, 1]

$$f(x) = 2^{|x|}, \qquad A = \int_{-1}^{1} 2^{|x|}\, dx = 2 \int_{0}^{1} 2^{x}\, dx = \frac{2}{\ln(2)}\, (2^{x})\Big|_{0}^{1} = \frac{2}{\ln(2)} \text{ FE}$$

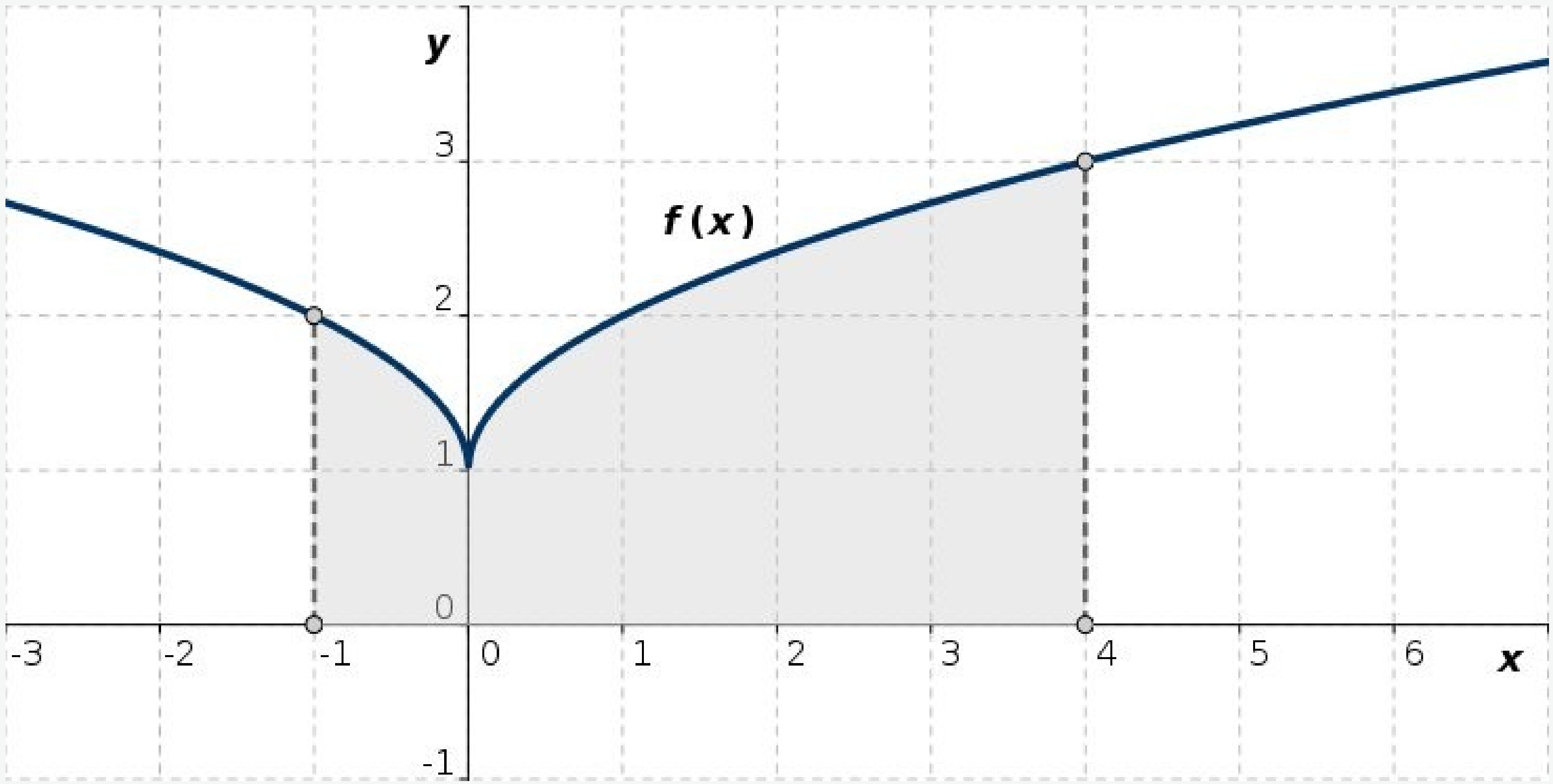


Abb. 1-6: Zur Berechnung der Fläche unter der Kurve y = f (x) im Intervall [-1, 4]

$$f(x) = \sqrt{|x|} + 1$$

$$A = \int_{-1}^{4} \left(\sqrt{|x|} + 1\right) dx = \int_{-1}^{0} \left(\sqrt{-x} + 1\right) + \int_{0}^{4} \left(\sqrt{x} + 1\right) =$$

$$= 2\int_{0}^{1} \left(\sqrt{x} + 1\right) + \int_{1}^{4} \left(\sqrt{x} + 1\right) = 11 \text{ FE}$$

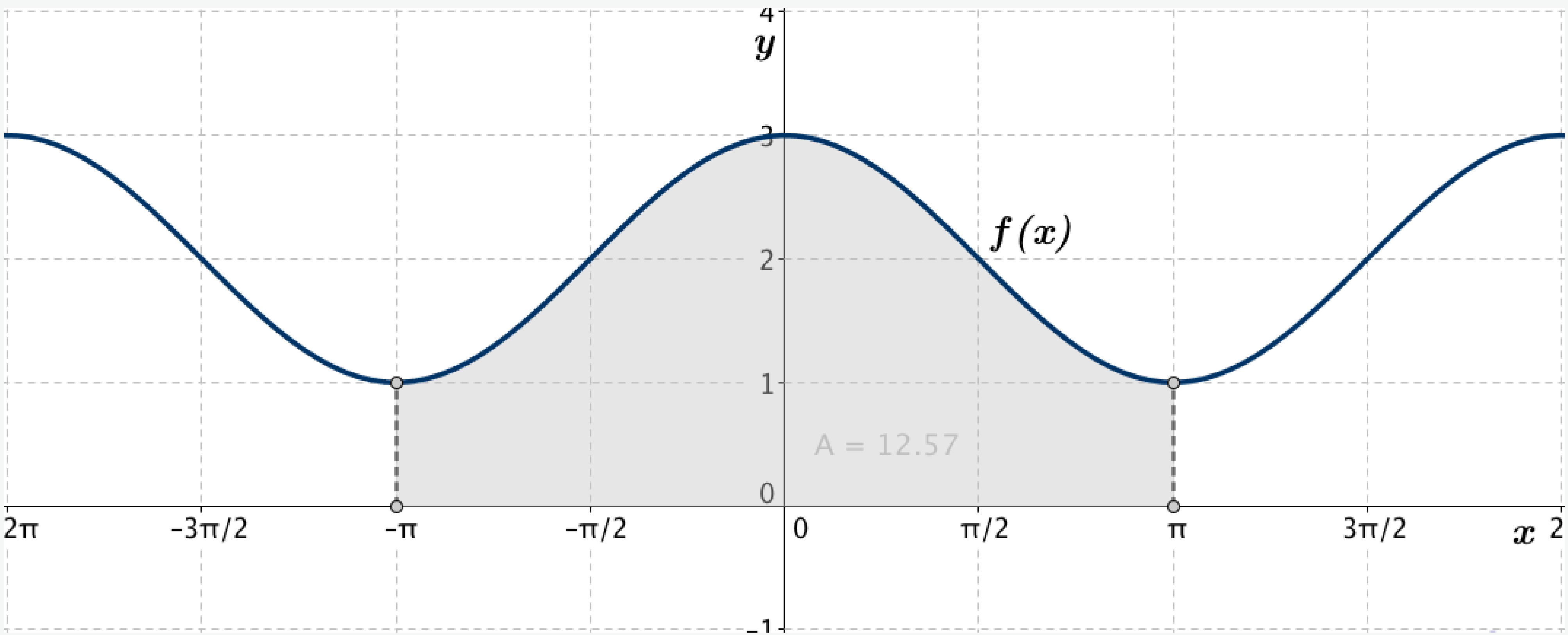


Abb. 1-7a: Zur Berechnung der Fläche unter der Kurve f (x) = cos x + 2 im Intervall [-π, π]

$$A = \int_{-\pi}^{\pi} (\cos x + 2)\ dx = (\sin x + 2\,x)\,\Big|_{-\pi}^{\pi} = 4\,\pi \simeq 12.57\ \text{FE}$$

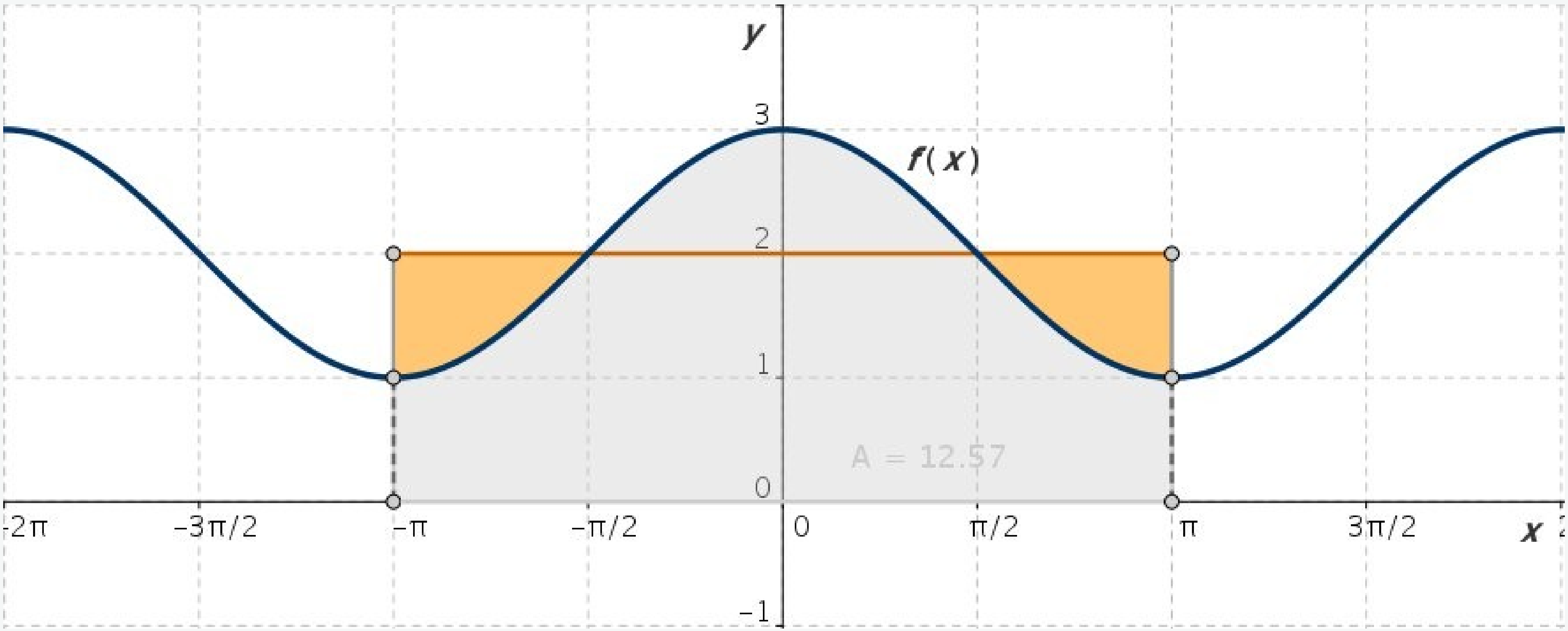


Abb. 1-7b: Zur Berechnung der Fläche unter der Kurve f (x) = cos x + 2 im Intervall [-π, π]

$$\int_{-\pi}^{\pi} \cos x \; dx = \sin x \,\Big|_{-\pi}^{\pi} = 0, \qquad 2 \int_{-\pi}^{\pi} dx = 2\, x \,\Big|_{-\pi}^{\pi} = 4\,\pi$$

$$A = \int_{-\pi}^{\pi} (\cos x + 2) \; dx = 2 \int_{-\pi}^{\pi} dx = 4\,\pi \simeq 12.57 \;\; \text{FE}$$

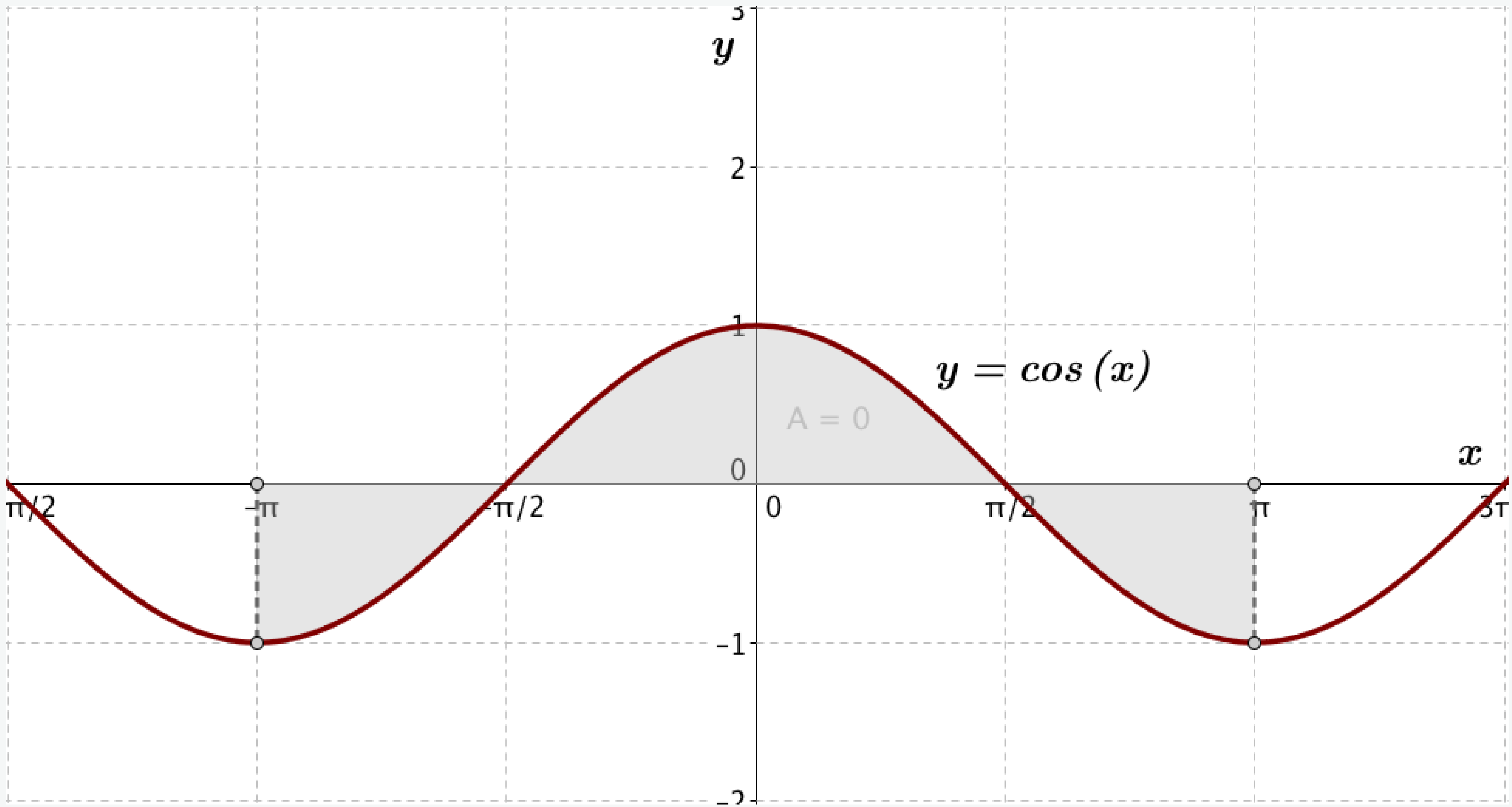


Abb. 1-7c: Das bestimmte Integral der trigonometrischer Funktion $y = \cos x$ über ihre Periode $T = 2\pi$

$$\int_{-\pi}^{\pi} \cos x \, dx = \sin x \,\Big|_{-\pi}^{\pi} = 0$$

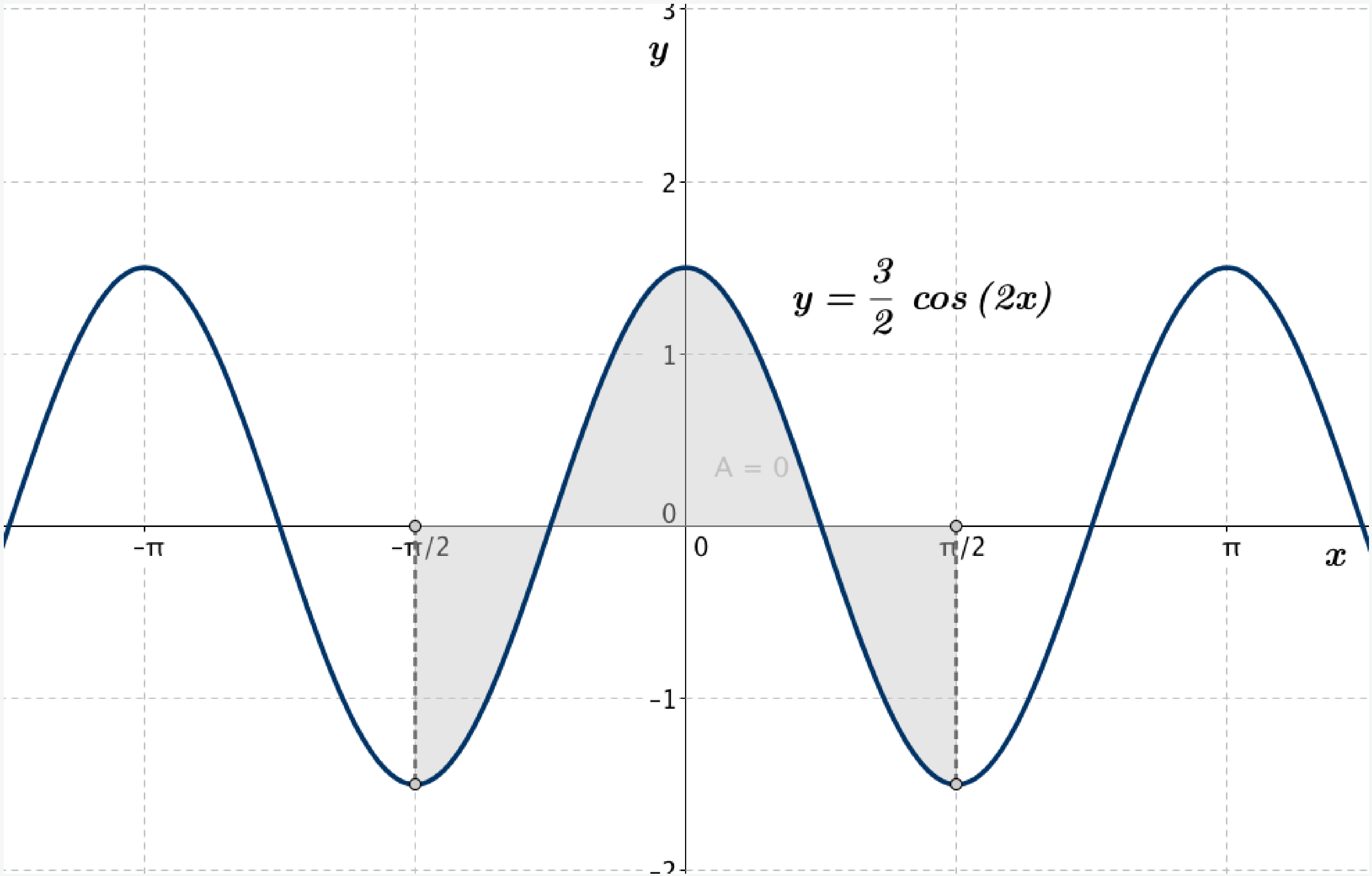


Abb. 1-7d: Das bestimmte Integral der trigonometrischer Funktion y = (3/2) cos (2x) über ihre Periode T = π

$$\int_{-\pi/2}^{\pi/2} \frac{3}{2}\cos(2x)\, dx = \frac{3}{4}\sin(2x)\Big|_{-\pi/2}^{\pi/2} = 0$$

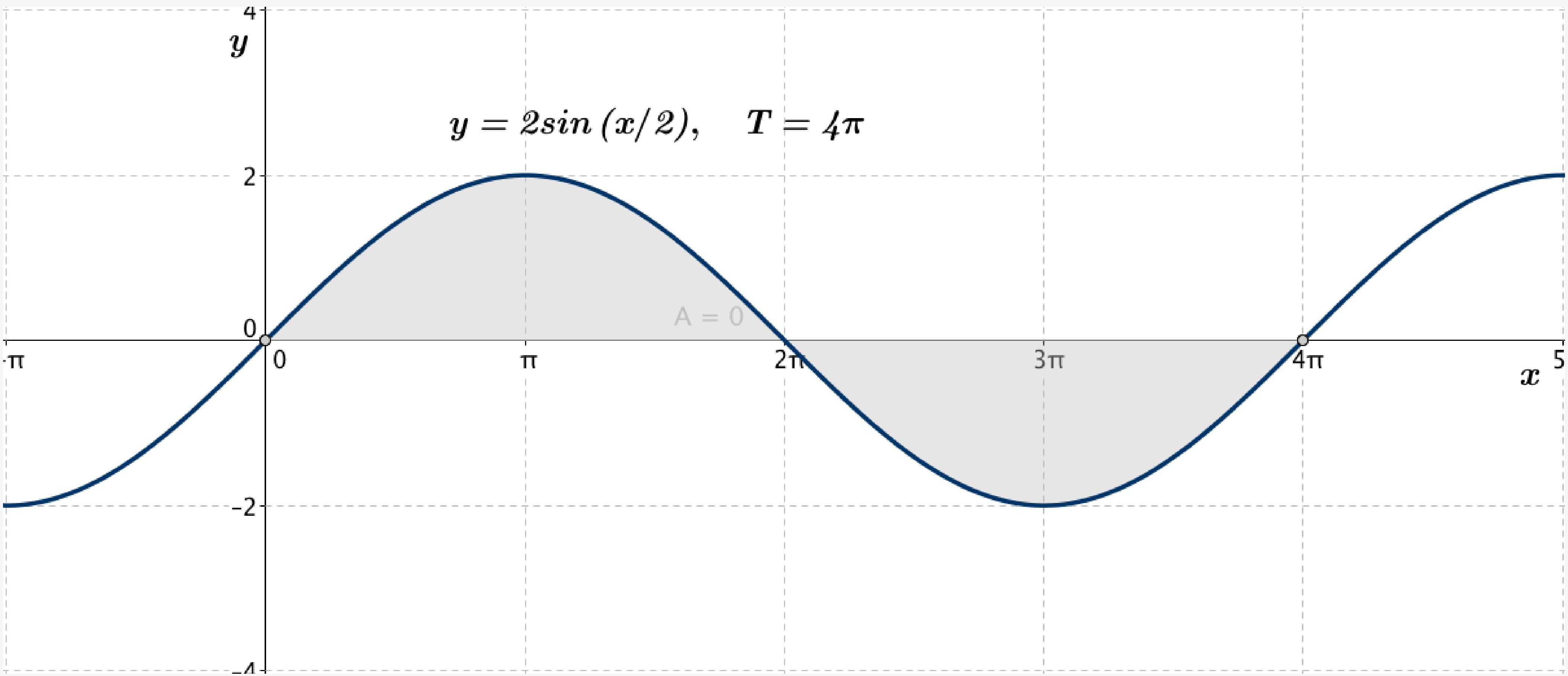


Abb. 1-7e: Das bestimmte Integral der trigonometrischer Funktion $y = 2\sin(x/2)$ über ihre Periode $T = 4\pi$

$$\int_0^{4\pi} 2\sin\left(\frac{x}{2}\right)\, dx = -4\cos\left(\frac{x}{2}\right)\Big|_0^{4\pi} = 0$$

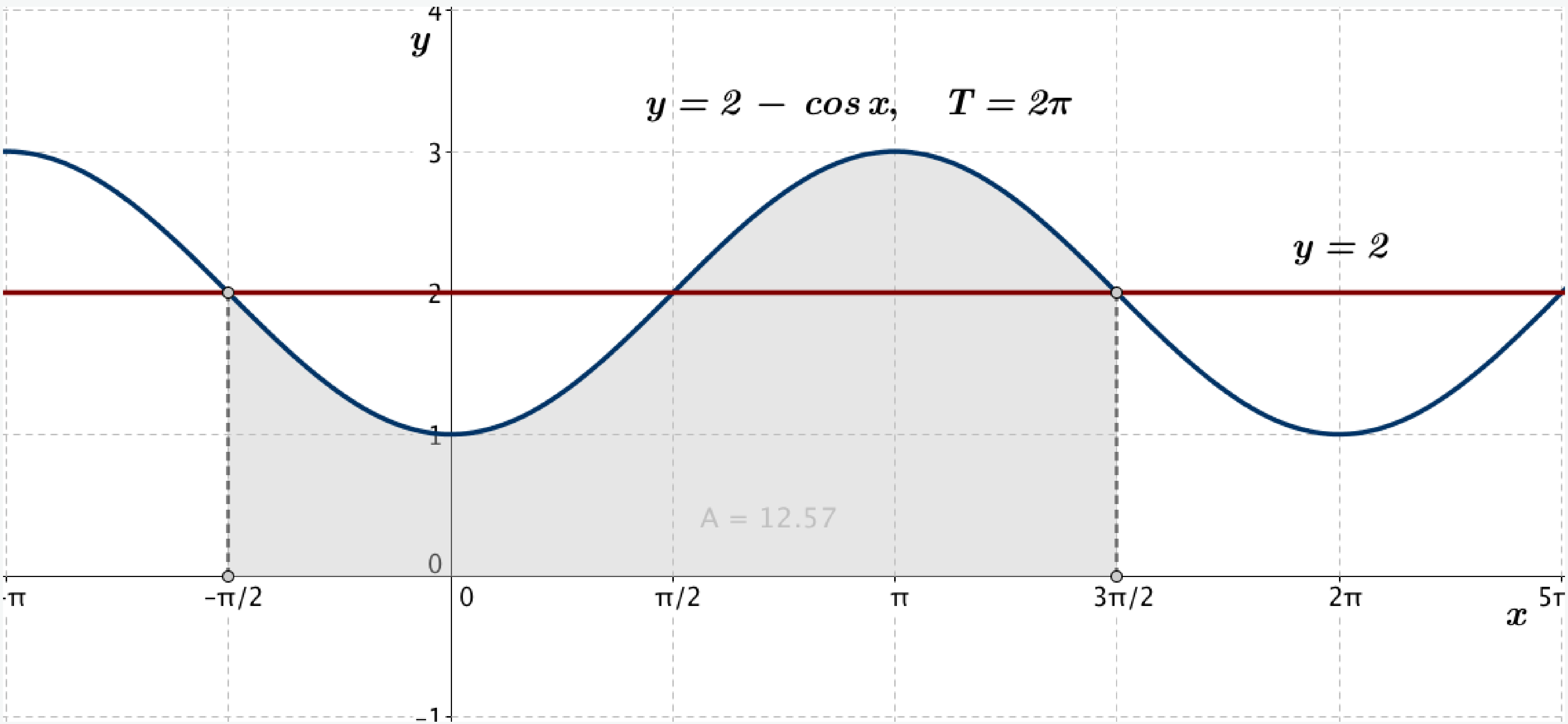


Abb. 1-8: Zur Berechnung der Fläche unter der Kurve f (x) = 2 - cos x im Intervall [-π/2, 3π/2]

$$A = \int_{-\pi/2}^{3\pi/2} (2 - \cos x)\, dx = (2x - \sin x)\Big|_{-\pi/2}^{3\pi/2} = 4\pi \simeq 12.57 \text{ FE}$$

$$A = \int_{-\pi/2}^{3\pi/2} (2 - \cos x)\, dx = 2 \int_{-\pi/2}^{3\pi/2} dx$$

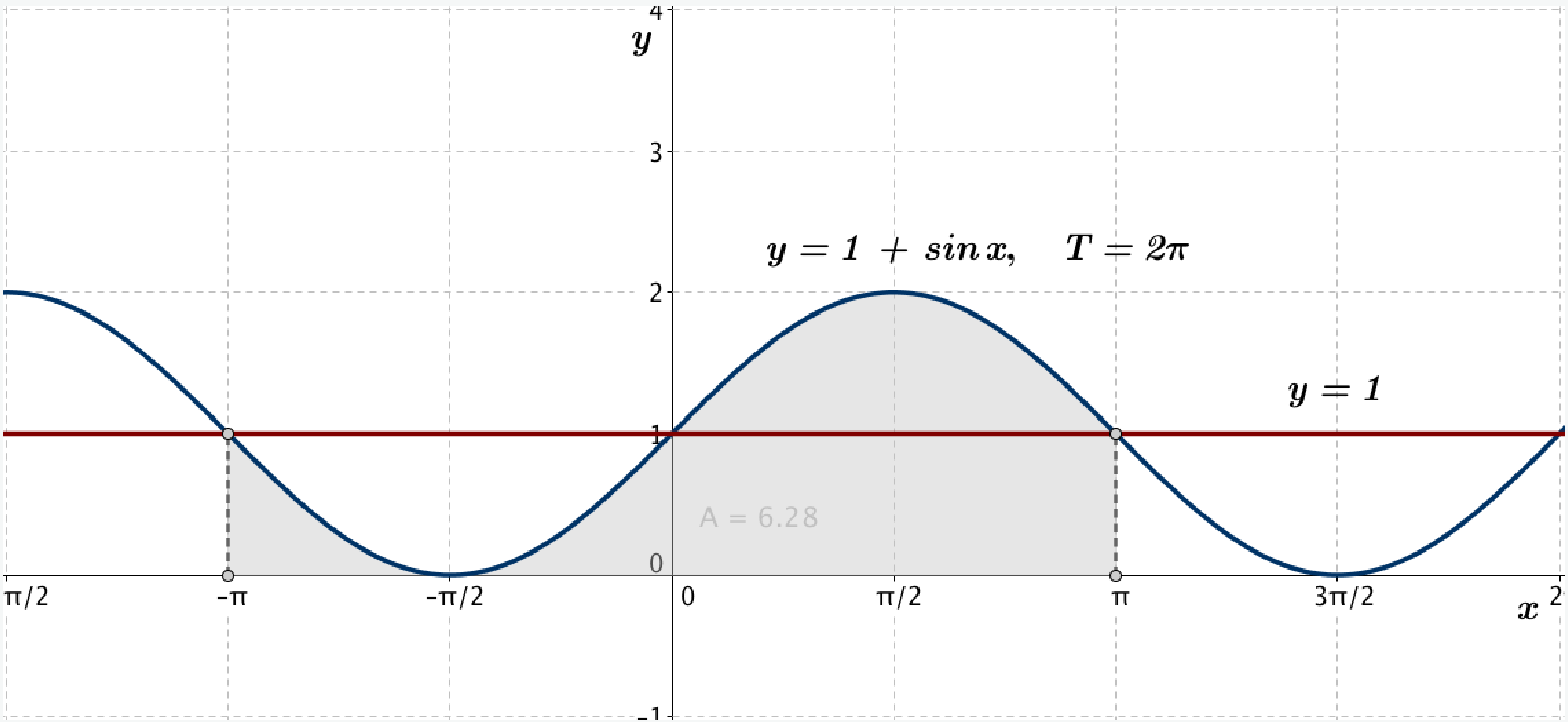


Abb. 1-9: Zur Berechnung der Fläche unter der Kurve f (x) = 1 + sin x im Intervall [-π, π]

$$A = \int_{-\pi}^{\pi} (1 + \sin x)\ dx = (x - \cos x)\Big|_{-\pi}^{\pi} = 2\pi \simeq 6.28\ \text{FE}$$

$$A = \int_{-\pi}^{\pi} (1 + \sin x)\ dx = \int_{-\pi}^{\pi} dx$$

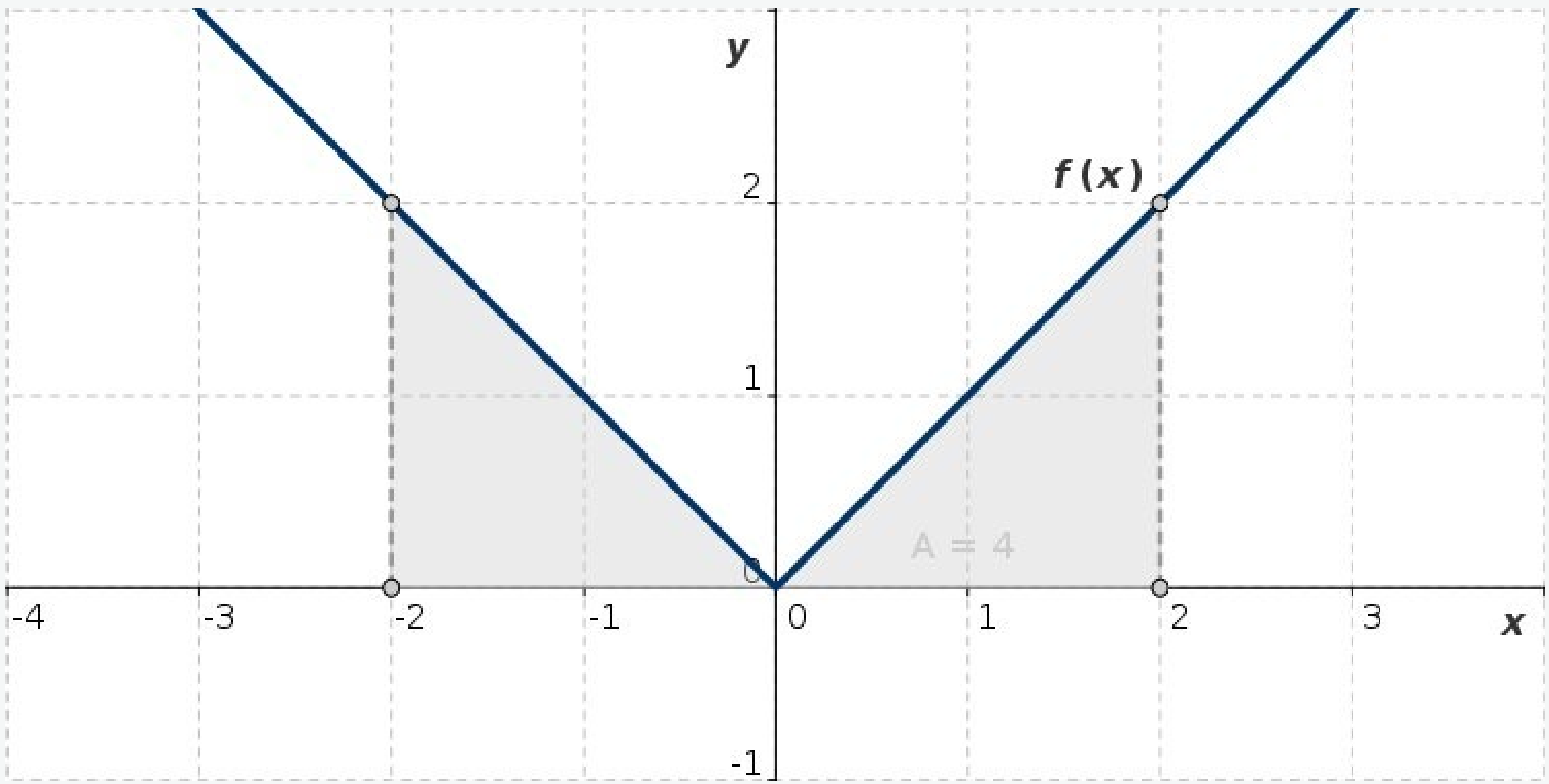


Abb. 1-10: Zur Berechnung der Fläche unter der Kurve f (x) = | x | im Intervall [-2, 2]

$$A = \int_{-2}^{2} |\, x \,|\; dx = 2 \int_{0}^{2} x\, dx = 4 \text{ FE}$$

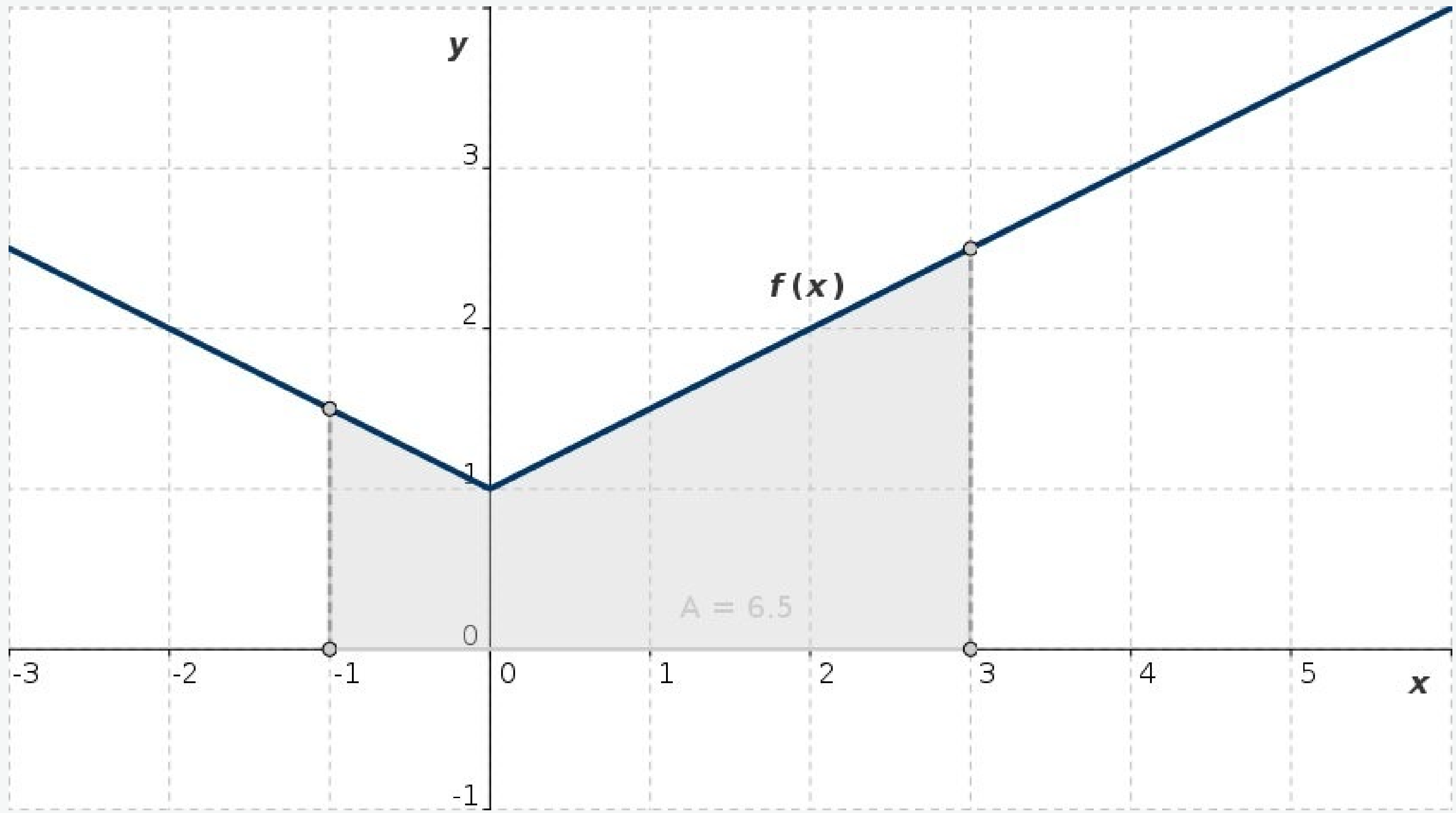
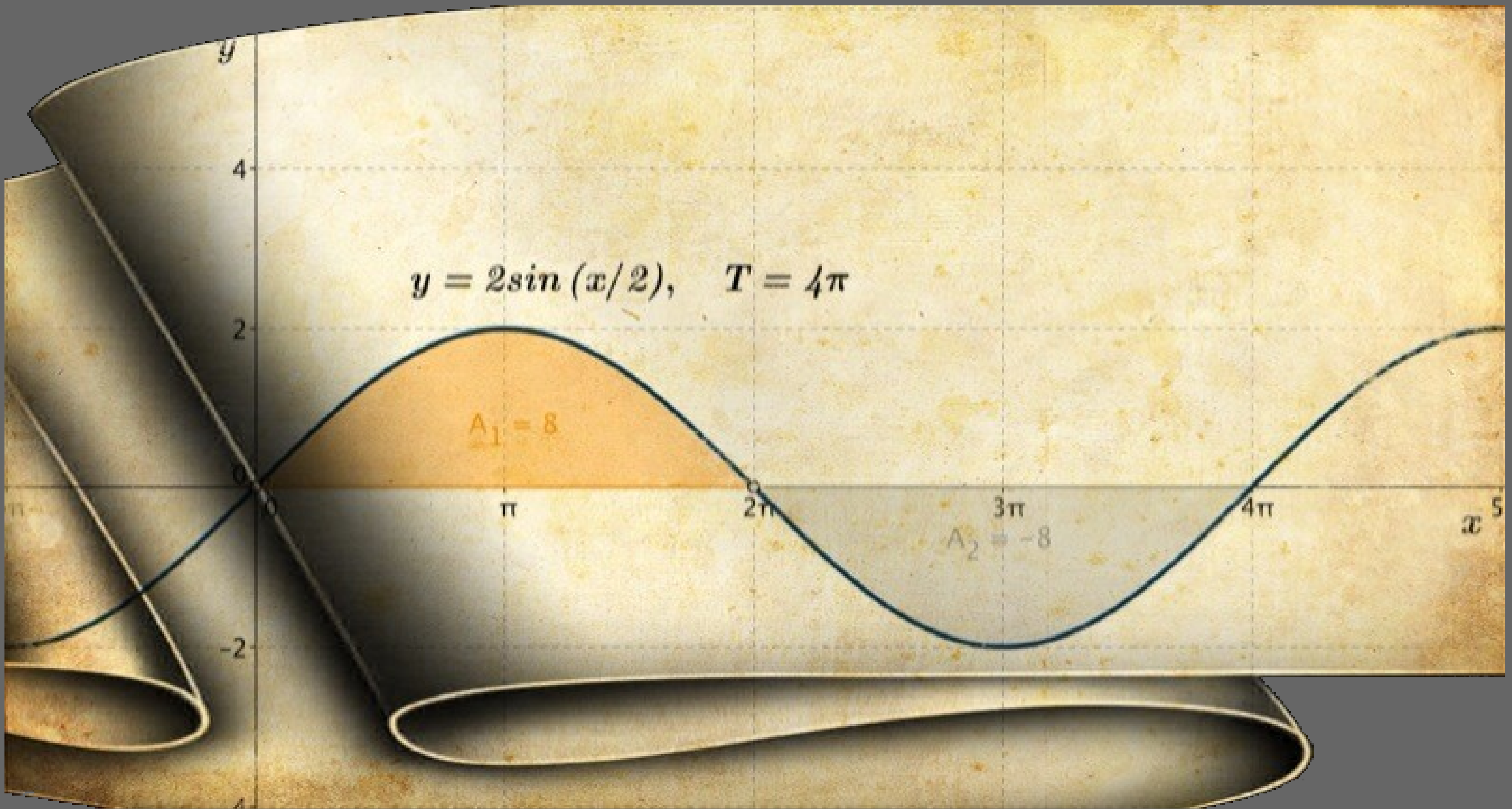


Abb. 1-11: Zur Berechnung der Fläche unter der Kurve f(x) = | x |/2 + 1 im Intervall [-1, 3]

$$A = \int_{-1}^{3} \left(\frac{|x|}{2} + 1 \right) dx = \int_{-1}^{0} \left(-\frac{x}{2} + 1 \right) dx + \int_{0}^{3} \left(\frac{x}{2} + 1 \right) dx = \frac{13}{2} \text{ FE}$$



$y = 2sin\,(x/2), \quad T = 4\pi$
$A_1 = 8$
$A_2 = -8$
y
x
4
2
0
-2
π
2π
3π
4π