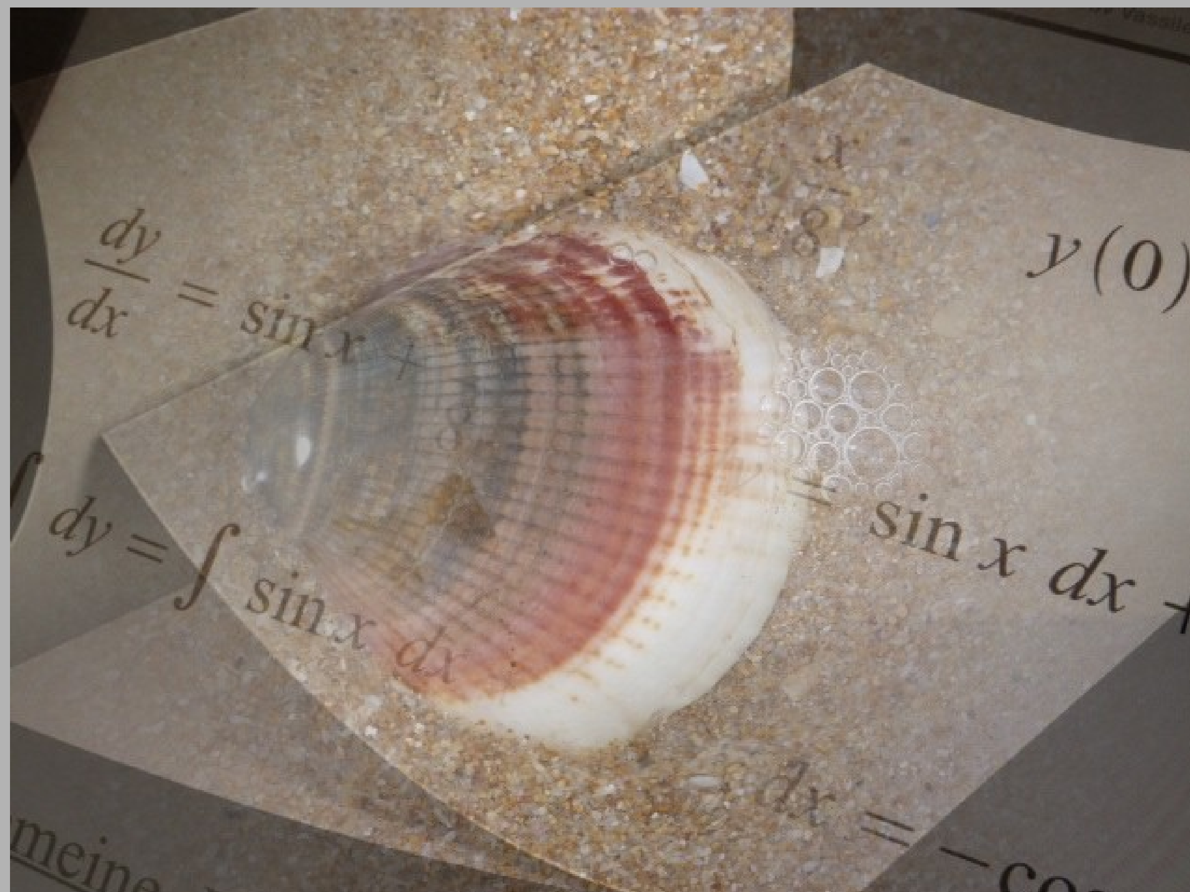


Gewöhnliche Differentialgleichungen: Aufgaben, Teil 2

Bestimmen Sie allgemeine und spezielle Lösungen der folgenden Differentialgleichungen

Aufgabe 11: $y' = 3x^2, \qquad y(-1) = 1$

Aufgabe 12: $y' + 2y = 0, \qquad y(0) = 2$

Aufgabe 13: $y' - y^2 = 0, \qquad y(1) = 2$

Aufgabe 14: $2y' - 5y^3 = 0, \qquad y(-1) = -\dfrac{1}{2}$

Aufgabe 15: $y' = \sin\left(\dfrac{x}{4}\right), \qquad y(2\pi) = -2$

Aufgabe 16: $y' = \cos(2x) - x$

1) $y(\pi) = -1, \qquad$ 2) $y(0) = -1,$

Aufgabe 17: $e^{y'} = e, \qquad y(-2) = 4$

Aufgabe 18: $e^{y'} = x^2, \qquad y(-1) = 3$

Aufgabe 19: $e^{y'} = \dfrac{1}{x}, \qquad y(1) = 2$

Aufgabe 20:

$$y''' = \sin x, \qquad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

Aufgabe 21: $y^{(4)} = x - \sin x$

Aufgabe 22: $x\, y^{(4)} = y'''$

Lösung 11: $y' = 3x^2, \quad y(-1) = 1$

AL: $y(x) = x^3 + C_1, \quad$ SL: $y(x) = x^3 + 2$

Lösung 12: $y' + 2y = 0, \quad y(0) = 2$

AL: $y(x) = C_1 e^{-2x}, \quad$ SL: $y(x) = 2e^{-2x}$

Lösung 13: $y' - y^2 = 0, \quad y(1) = 2$

AL: $y(x) = \dfrac{1}{C_1 - x}, \quad$ SL: $y(x) = \dfrac{2}{3 - 2x}$

Lösung 14: $2y' - 5y^3 = 0, \quad y(-1) = -\dfrac{1}{2}$

AL: $y(x) = -\dfrac{1}{\sqrt{C_1 - 5x}}, \quad$ SL: $y(x) = -\dfrac{1}{\sqrt{-5x - x}}$

Lösung 15: $y' = \sin\left(\dfrac{x}{4}\right), \quad y(2\pi) = -2$

AL: $y(x) = C_1 - 4\cos\left(\dfrac{x}{4}\right), \quad$ SL: $y(x) = -2\left(2\cos\left(\dfrac{x}{4}\right) + 1\right)$

$$y' = \cos(2x) - x$$

$$1)\ y(\pi) = -1, \qquad 2)\ y(0) = -1, \qquad 3)\ y(0) = \frac{1}{2}$$

$$\text{AL:} \quad y(x) = \frac{\sin(2x)}{2} - \frac{x^2}{2} + C_1$$

$$1)\ \text{SL:} \quad y(\pi) = -1, \qquad y_1(x) = \frac{\sin(2x)}{2} - \frac{x^2}{2} + \frac{\pi^2}{2} - 1$$

$$2)\ \text{SL:} \quad y(0) = -1, \qquad y_2(x) = \frac{\sin(2x)}{2} - \frac{x^2}{2} - 1$$

$$3)\ \text{SL:} \quad y(0) = \frac{1}{2}, \qquad y_3(x) = \frac{\sin(2x)}{2} - \frac{x^2}{2} + \frac{1}{2}$$

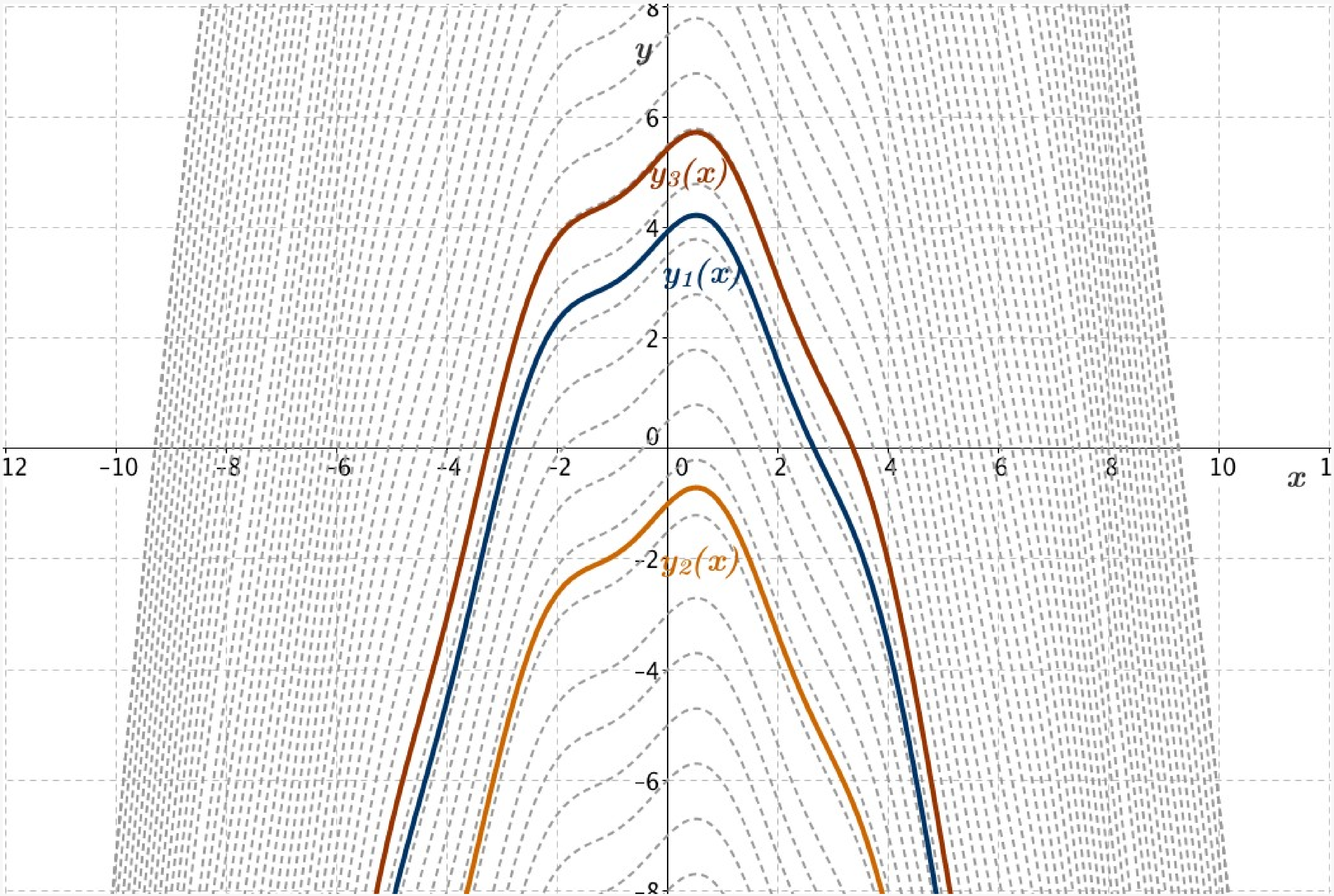


Abb. L16: Integralkurven der DGL y' (x) = cos (2x) - x

Lösung 16: $y' = \cos(2x) - x, \quad y(\pi) = 3$

AL: $y(x) = \dfrac{\sin(2x)}{2} - \dfrac{x^2}{2} + C_1, \quad$ SL: $y(x) = \dfrac{\sin(2x)}{2} - \dfrac{x^2}{2} + \dfrac{\pi^2}{2} + 3$

Lösung 17: $e^{y'} = e, \quad y(-2) = 4$

AL: $y(x) = x + C_1, \quad$ SL: $y(x) = x + 1$

Lösung 18: $e^{y'} = x^2, \quad y(-1) = 3$

AL: $y(x) = -2x + x\ln(x^2) + C_1, \quad$ SL: $y(x) = -2x + x\ln(x^2) + 1$

Lösung 19: $e^{y'} = \dfrac{1}{x}, \quad y(1) = 2$

AL: $y(x) = x\,(1 - \ln(x)) + C_1, \quad$ SL: $y(x) = x\,(1 - \ln(x)) + 1$

Lösung 20: $y''' = x - \sin x, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$

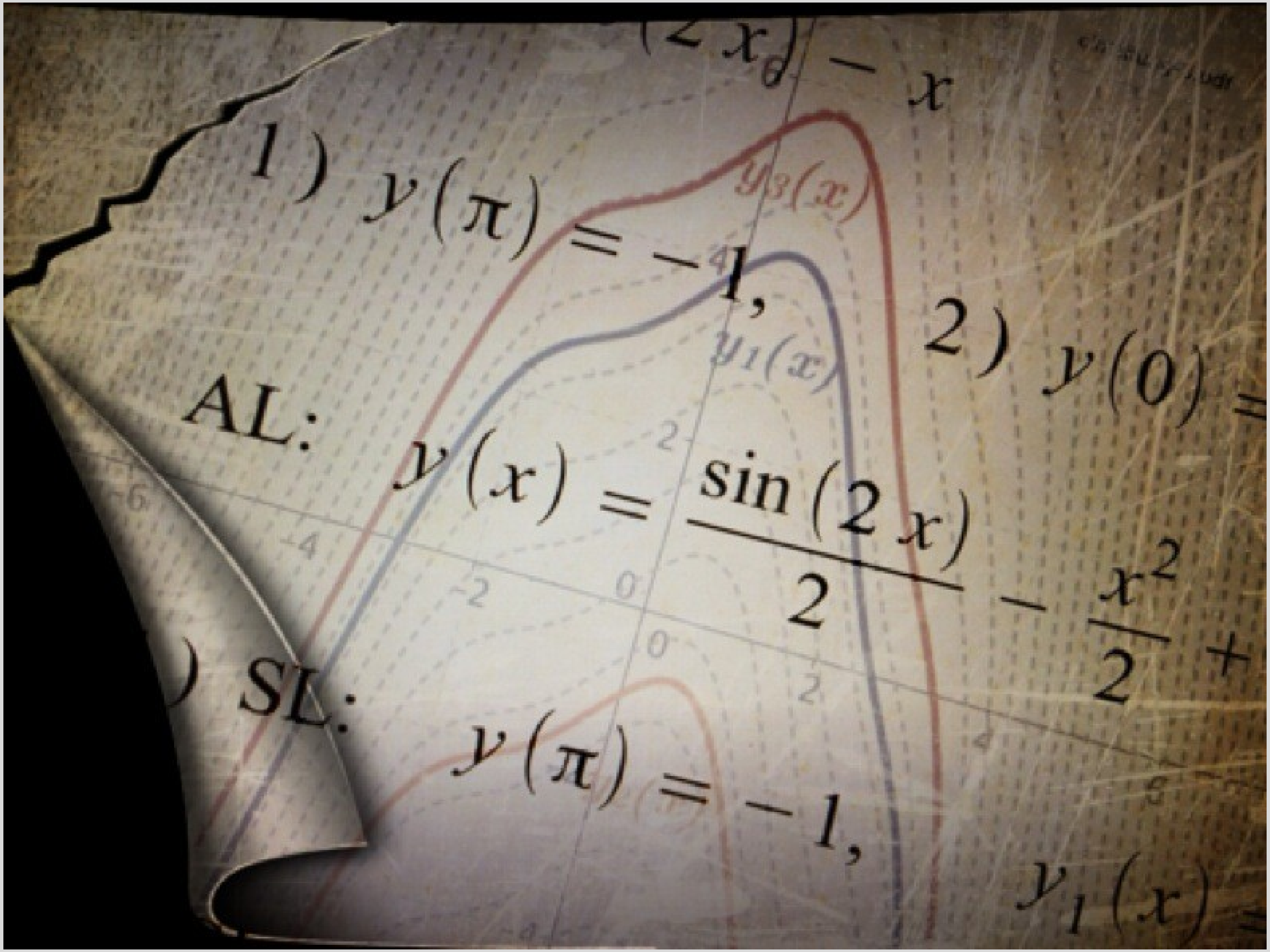
AL: $y(x) = \cos x + C_3 x^2 + C_2 x + C_1, \quad$ SL: $y(x) = 2x(x+1) + \cos x$

Lösung 21: $y^{(4)} = x - \sin x$

AL: $y(x) = -\sin x + \dfrac{x^5}{120} + C_4 x^3 + C_3 x^2 + C_2 x + C_1$

Lösung 22: $x\, y^{(4)} = y''', \qquad p = y''', \qquad x \dfrac{dp}{dx} = p$

AL: $y(x) = C_1 x^4 + C_2 x^2 + C_3 x + C_4$


$1)\ y(\pi) = -1,$
$2)\ y(0) =$
$y_3(x)$
$y_1(x)$
AL: $y(x) = \frac{\sin(2x)}{2} - \frac{x^2}{2} +$
SL: $y(\pi) = -1,$
$y_1(x) =$