Bestimmen Sie die Lösungen folgender komplexen Gleichungen:

Aufgabe 5:

$a)\ z^3 + 8 = 0, \quad b)\ z^4 + 8 = 0, \quad c)\ z^5 + 8 = 0$

Aufgabe 6:

$a)\ z^3 = -1 + i, \quad b)\ z^4 = -1 + i$

Aufgabe 7:

$a)\ z^4 = 2 + 2i, \quad b)\ z^8 = 2 + 2i$

$$z^3 + 8 = 0$$

$$z^3 = -8 = 8\, e^{i\pi} = 8\,(\cos\pi + i\sin\pi)$$

$$W_k = \sqrt[3]{8}\; e^{i\,\frac{\pi + 2\pi k}{3}} = \sqrt[3]{8}\left(\cos\left(\frac{\pi}{3}(1 + 2k)\right) + i\sin\left(\frac{\pi}{3}(1 + 2k)\right)\right)$$

$$k = 0,\ 1,\ 2$$

$$W_0 = 1 + i\sqrt{3}\,, \qquad W_1 = -2, \qquad W_2 = 1 - i\sqrt{3}$$

$$W_2 = W_0^*, \qquad W_1 = W_1^*$$

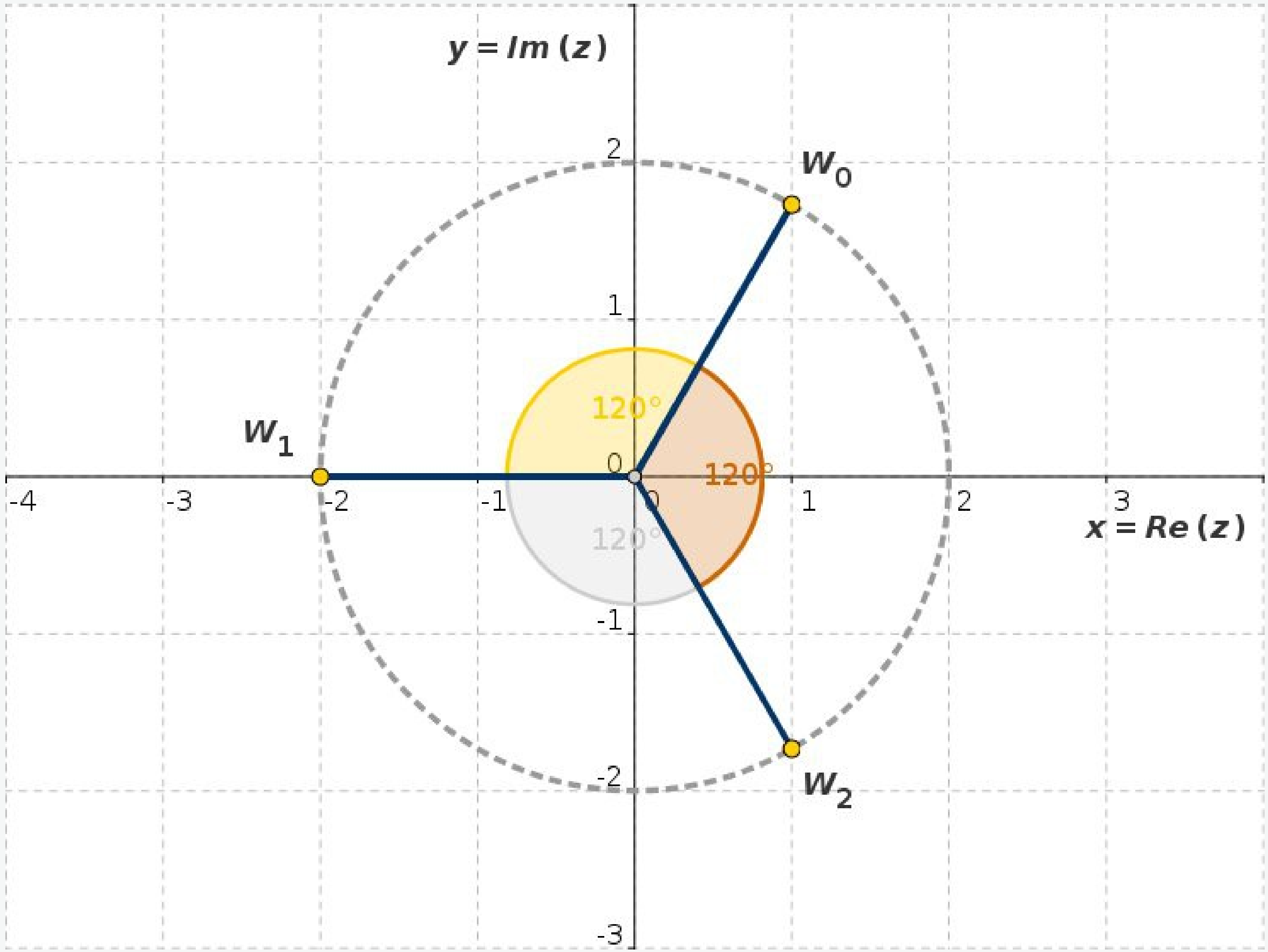

Abb. L5a: Graphische Darstellung der komplexen Lösungen der Gleichung

Wurzelziehen: Lösung 5b

$$z^4 + 8 = 0$$

$$z^4 = -8 = 8\, e^{i\pi} = 8\,(\cos\pi + i\sin\pi)$$

$$W_k = \sqrt[4]{8}\; e^{i\frac{\pi + 2\pi k}{4}} = \sqrt[4]{8}\left(\cos\left(\frac{\pi}{4}(1 + 2k)\right) + i\sin\left(\frac{\pi}{4}(1 + 2k)\right)\right)$$

$$k = 0,\ 1,\ 2,\ 3$$

$$W_0 = \sqrt[4]{2}\,(1 + i), \qquad W_1 = \sqrt[4]{2}\,(-1 + i)$$

$$W_2 = -\sqrt[4]{2}\,(1 + i), \qquad W_3 = \sqrt[4]{2}\,(1 - i)$$

$$W_3 = W_0^*, \qquad W_2 = W_1^*$$

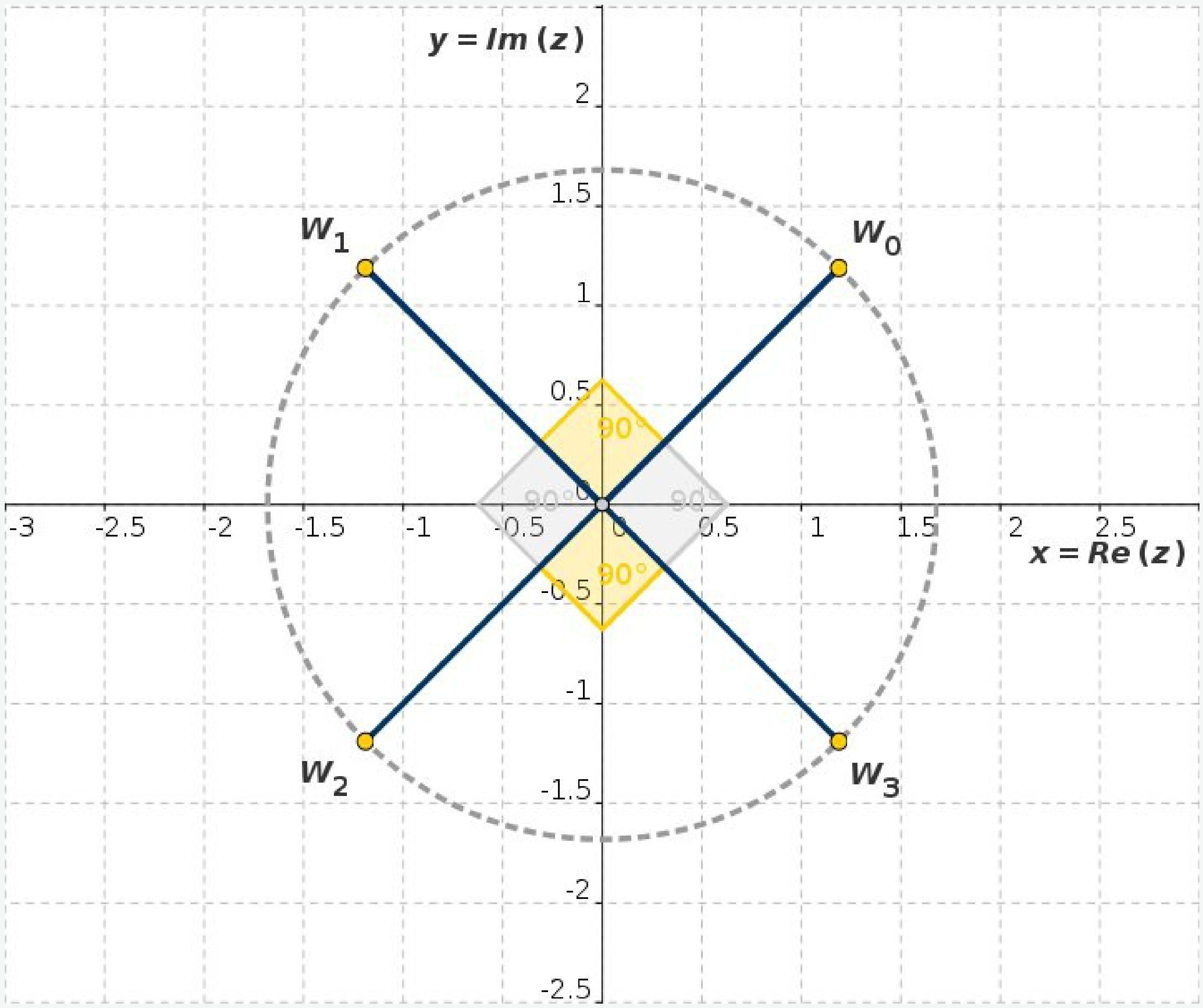

Abb. L5b: Graphische Darstellung der komplexen Lösungen der Gleichung

$$z^5 + 8 = 0, \qquad z^5 = -8 = 8\, e^{i\pi} = 8\,(\cos\pi + i\sin\pi)$$

$$W_k = \sqrt[5]{8}\; e^{\,i\frac{\pi + 2\pi k}{5}} = \sqrt[5]{8}\left(\cos\left(\frac{\pi}{5}(1 + 2k)\right) + i\sin\left(\frac{\pi}{5}(1 + 2k)\right)\right)$$

$$k = 0,\ 1,\ 2,\ 3,\ 4$$

$$W_0 = 2^{\frac{3}{5}}\left(\cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)\right) \simeq 1.226 + i\,0.891$$

$$W_1 = 2^{\frac{3}{5}}\left(-\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right) \simeq -0.468 + i\,1.442$$

$$W_2 = -2^{\frac{3}{5}} \simeq -1.516$$

$$W_3 = -2^{\frac{3}{5}}\left(\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right) \simeq -0.468 - i\,1.442$$

$$W_4 = 2^{\frac{3}{5}}\left(\cos\left(\frac{\pi}{5}\right) - i\sin\left(\frac{\pi}{5}\right)\right) \simeq 1.226 - i\,0.891$$

$$W_4 = W_0^*, \qquad W_3 = W_1^*, \qquad W_2 = W_2^*$$

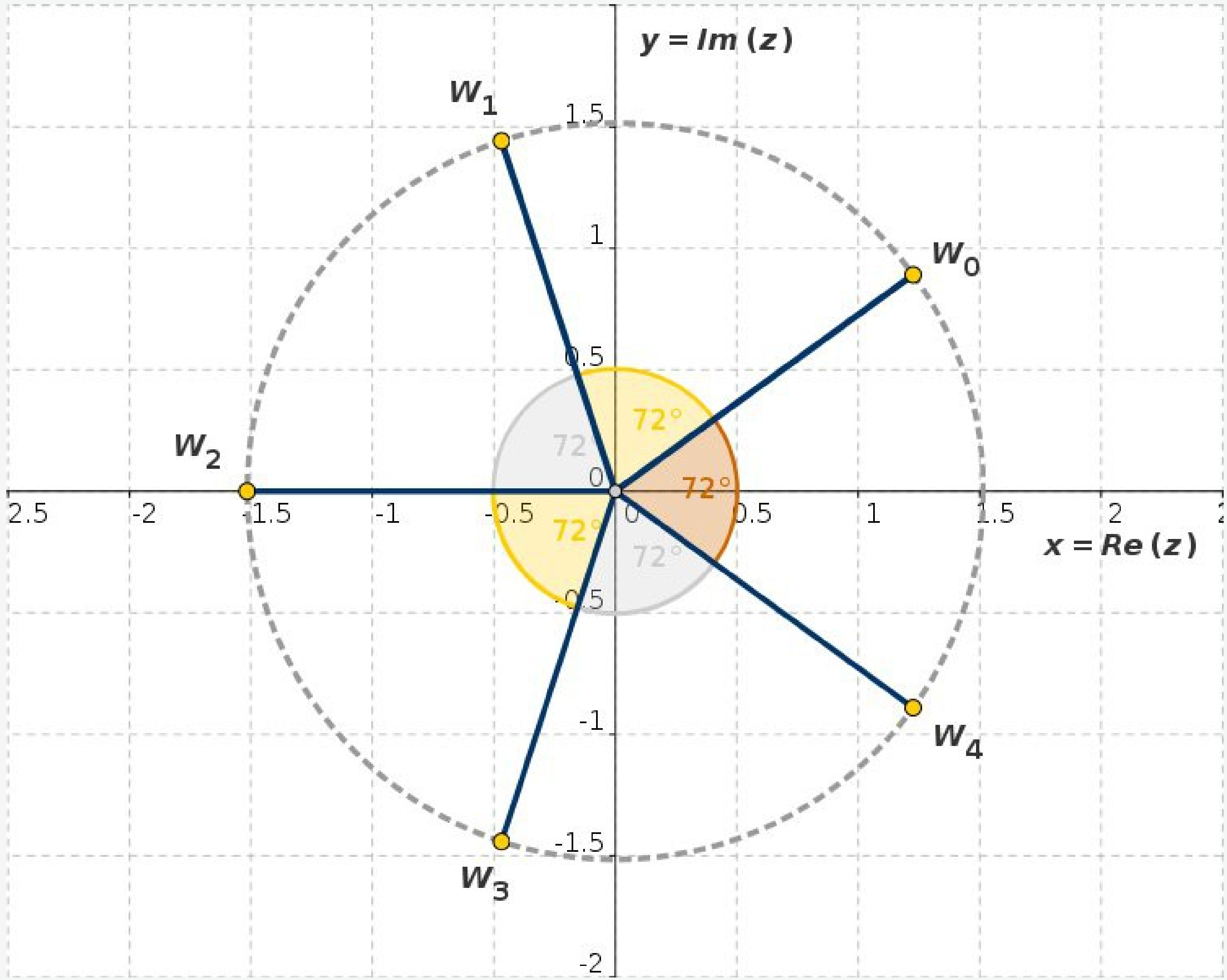

Abb. L5c: Graphische Darstellung der komplexen Lösungen der Gleichung

$$z^3 = -1 + i = \sqrt{2}\; e^{i \frac{3\pi}{4}} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$$

$$W_k = \sqrt[6]{2}\;\; e^{i\pi\left(\frac{1}{4} + \frac{2k}{3}\right)} = \sqrt[6]{2}\left(\cos\left(\frac{\pi}{4} + \frac{2k}{3}\pi\right) + i \sin\left(\frac{\pi}{4} + \frac{2k}{3}\pi\right)\right)$$

$$k = 0,\ 1,\ 2$$

$$W_0 = \frac{1 + i}{\sqrt[3]{2}} \simeq 0.798\,(1 + i)$$

$$W_1 = \sqrt[6]{2}\left(-\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right)\right) \simeq -1.084 + i\, 0.291$$

$$W_2 = \sqrt[6]{2}\left(\cos\left(\frac{5\pi}{12}\right) - i \sin\left(\frac{5\pi}{12}\right)\right) \simeq 0.291 - i\, 1.084$$

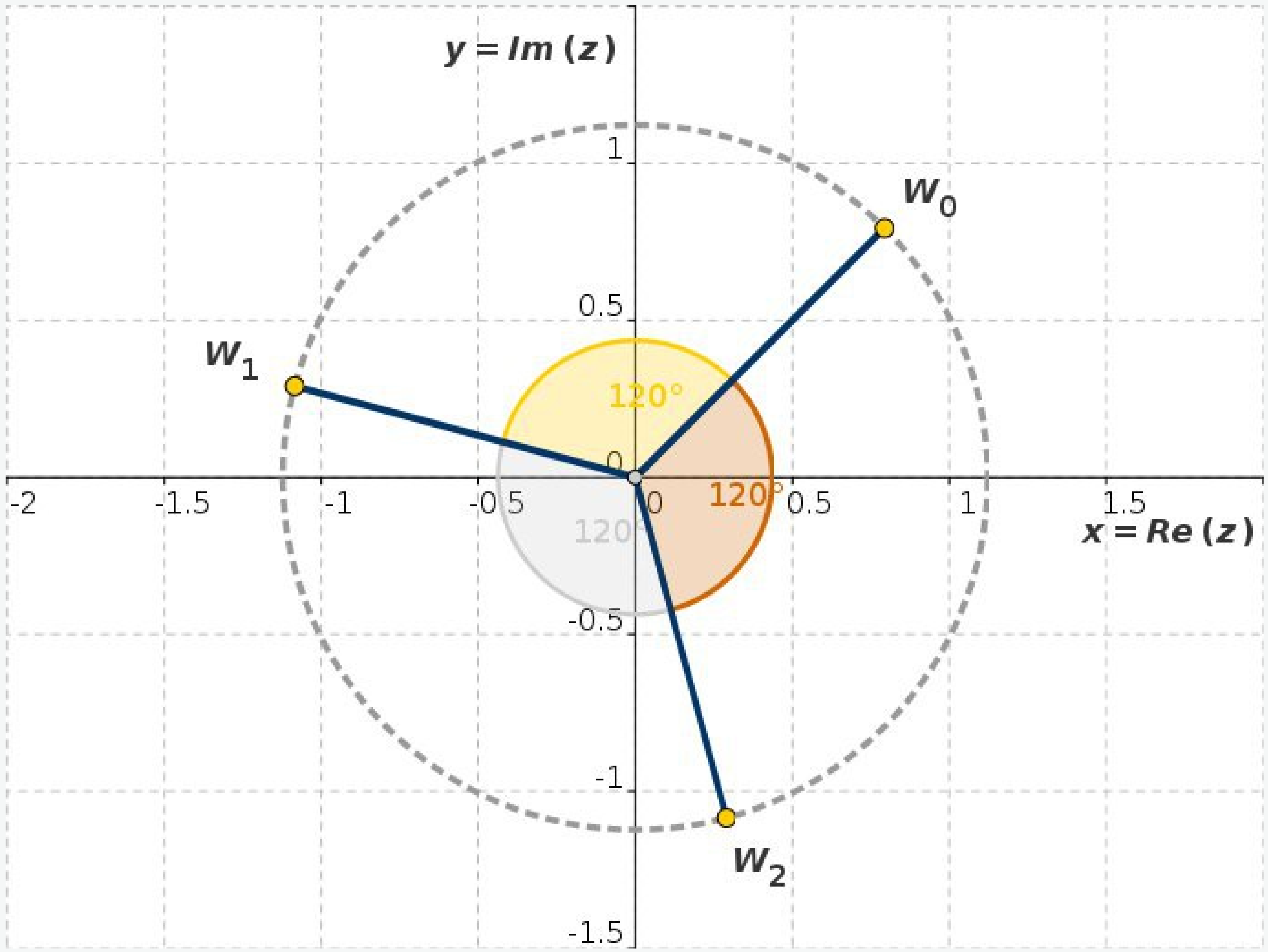

Abb. L6a: Graphische Darstellung der komplexen Lösungen der Gleichung

$$z^4 = -1 + i = \sqrt{2}\; e^{i\,\frac{3\pi}{4}} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$W_k = \sqrt[8]{2}\; e^{i\pi\left(\frac{3}{16}+\frac{k}{2}\right)} = \sqrt[8]{2}\left(\cos\left(\frac{3\pi}{16}+\frac{k\pi}{2}\right) + i\sin\left(\frac{3\pi}{16}+\frac{k\pi}{2}\right)\right)$$

$$k = 0,\ 1,\ 2,\ 3$$

$$W_0 = \sqrt[8]{2}\left(\cos\left(\frac{3\pi}{16}\right) + i\sin\left(\frac{3\pi}{16}\right)\right) \simeq 0.907 + i\,0.606$$

$$W_1 = \sqrt[8]{2}\left(\cos\left(\frac{3\pi}{16}+\frac{\pi}{2}\right) + i\sin\left(\frac{3\pi}{16}+\frac{\pi}{2}\right)\right) \simeq -0.606 + i\,0.907$$

$$W_2 = \sqrt[8]{2}\left(\cos\left(\frac{3\pi}{16}+\pi\right) + i\sin\left(\frac{3\pi}{16}+\pi\right)\right) \simeq -0.907 - i\,0.606$$

$$W_3 = \sqrt[8]{2}\left(\cos\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right)\right) \simeq 0.606 - i\,0.907$$

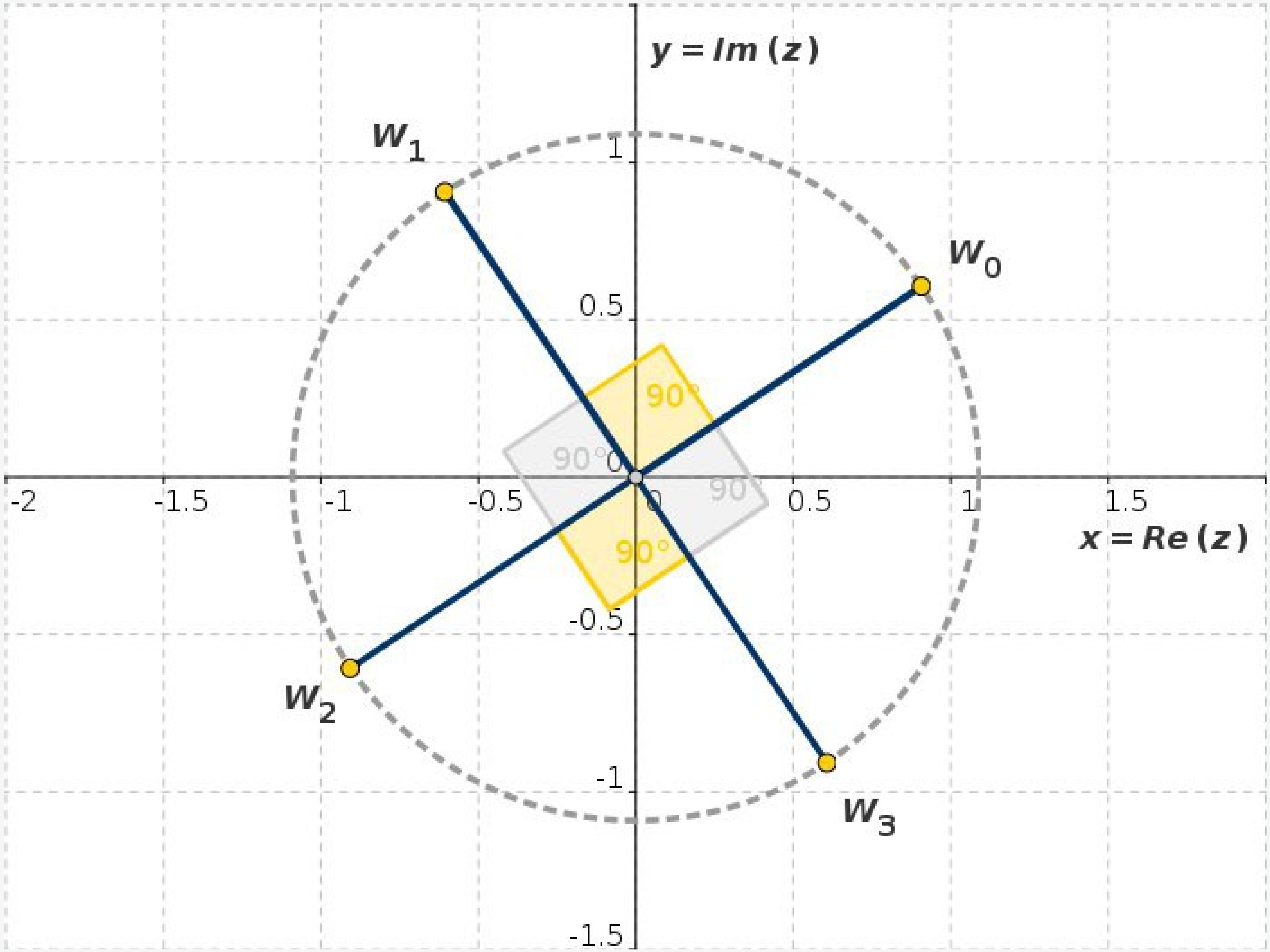

Abb. L6b: Graphische Darstellung der komplexen Lösungen der Gleichung

$$z^4 = 2 + 2i = 2\sqrt{2}\, e^{i\frac{\pi}{4}} = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$W_k = 2^{\frac{3}{8}}\, e^{i\pi\left(\frac{1}{16}+\frac{k}{2}\right)} = 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16}+\frac{k\pi}{2}\right) + i\sin\left(\frac{\pi}{16}+\frac{k\pi}{2}\right)\right)$$

$k = 0,\ 1,\ 2,\ 3$

$$W_0 = 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16}\right) + i\sin\left(\frac{\pi}{16}\right)\right) \simeq 1.272 + i\,0.253$$

$$W_1 = 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16}+\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{16}+\frac{\pi}{2}\right)\right) \simeq -0.253 + i\,1.272$$

$$W_2 = 2^{\frac{3}{8}}\left(\cos\left(\frac{\pi}{16}+\pi\right) + i\sin\left(\frac{\pi}{16}+\pi\right)\right) \simeq -1.272 - i\,0.253$$

$$W_3 = 2^{\frac{3}{8}}\left(\cos\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{16}+\frac{3\pi}{2}\right)\right) \simeq 0.253 - i\,1.272$$

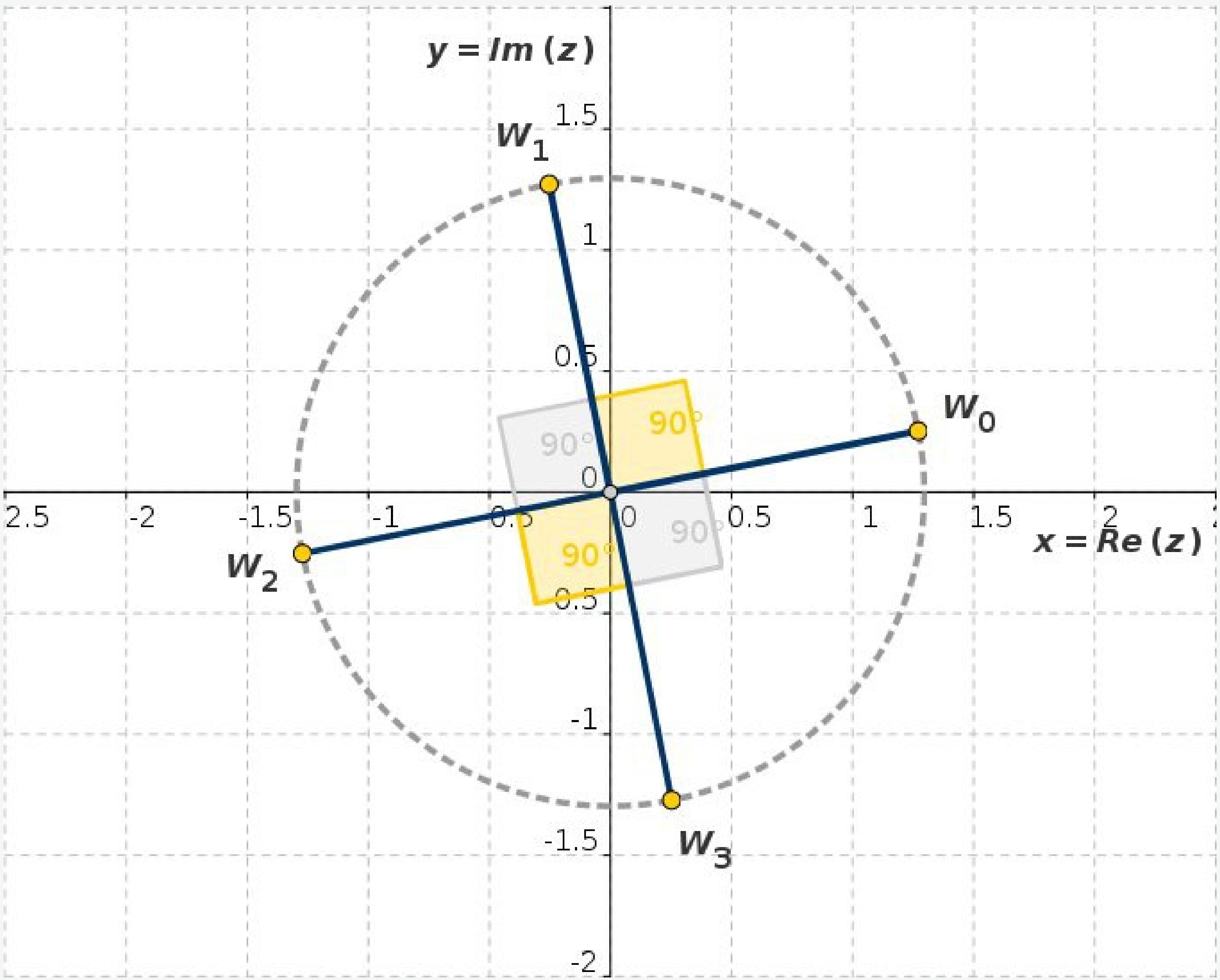

Abb. L7a: Graphische Darstellung der komplexen Lösungen der Gleichung

Wurzelziehen: Lösung 7b

$$z^8 = 2 + 2i = 2\sqrt{2}\, e^{i\frac{\pi}{4}} = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$W_k = 2^{\frac{3}{16}}\, e^{i\pi\left(\frac{1}{32} + \frac{k}{4}\right)} = 2^{\frac{3}{16}}\left(\cos\left(\frac{\pi}{32} + \frac{k\pi}{4}\right) + i\sin\left(\frac{\pi}{32} + \frac{k\pi}{4}\right)\right)$$

$$k = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$$

$$W_0 \simeq 1.133 + i\,0.112, \qquad W_1 \simeq 0.722 + i\,0.880$$

$$W_2 \simeq -0.112 + i\,1.133, \qquad W_3 \simeq -0.880 + i\,0.722$$

$$W_4 \simeq -1.133 - i\,0.112, \qquad W_5 \simeq -0.722 - i\,0.880$$

$$W_6 \simeq 0.112 - i\,1.133, \qquad W_7 \simeq 0.880 - i\,0.722$$

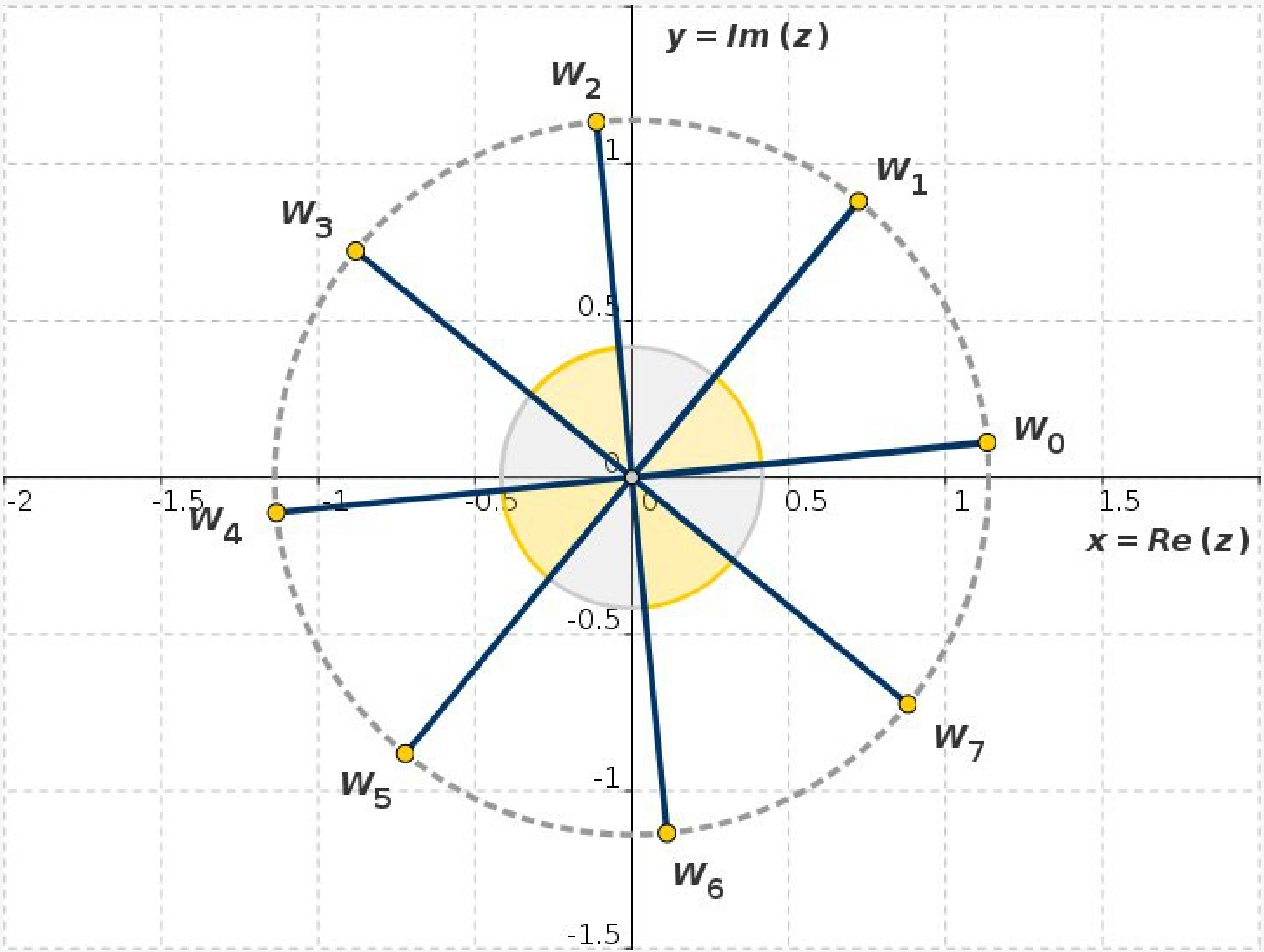

Abb. L7b: Graphische Darstellung der komplexen Lösungen der Gleichung