Kartesische Form, Polarform: Umrechnung

Die in kartesischer Form gegebenen komplexen Zahlen sind in Polarform umzurechnen

Aufgabe 1: $z = \sqrt{2} + \sqrt{2}\, i$

Aufgabe 2: $z = -3 + 5\, i$

Aufgabe 3: $z = -\dfrac{3\sqrt{3}}{2} - \dfrac{3}{2}\, i$

Aufgabe 4: $z = 1 - i$

Aufgabe 5: $z = 2\, i$

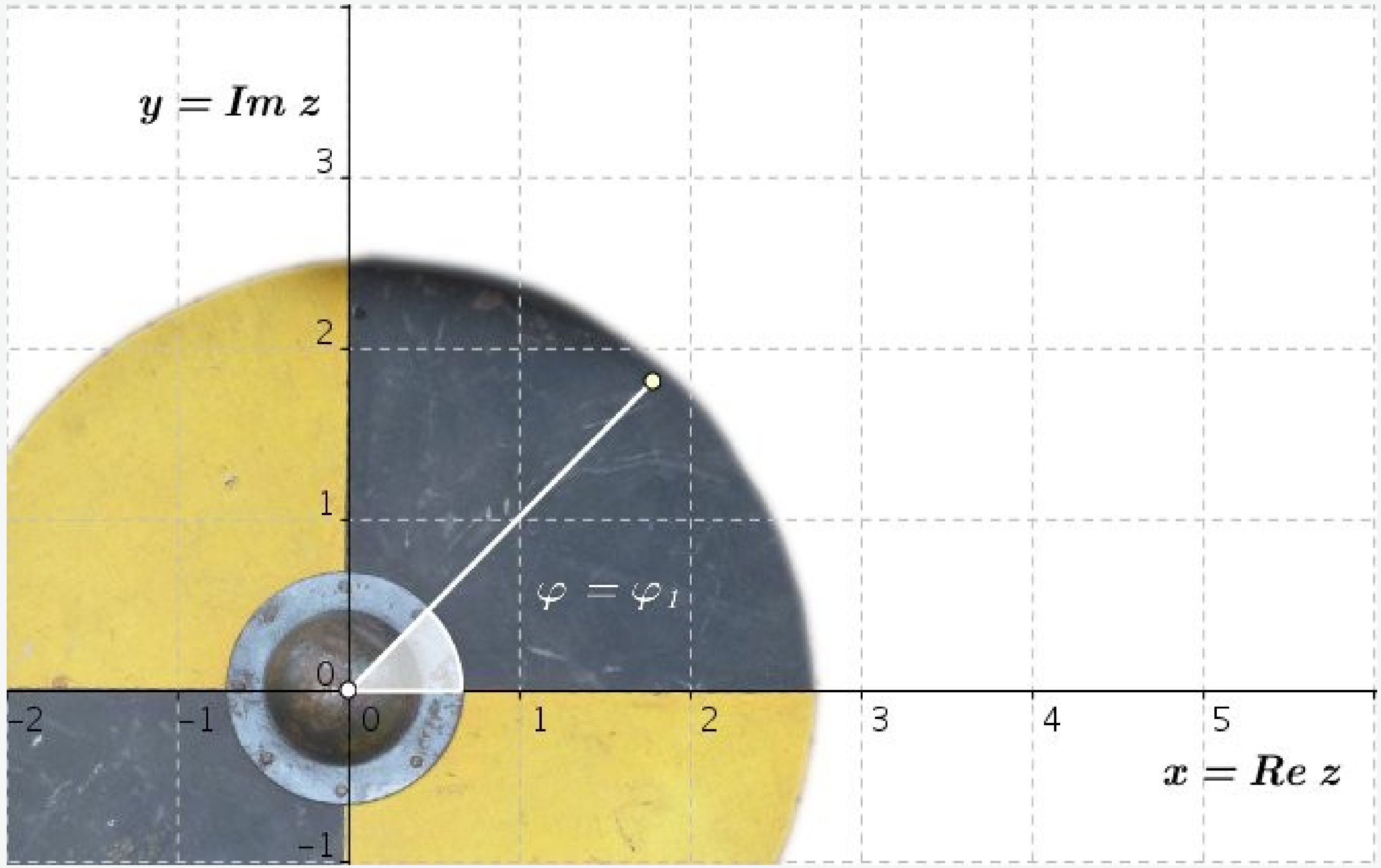

Abb. 5-1: Zur Bestimmung des Polarwinkels im ersten Quadranten

$$z = \sqrt{2} + \sqrt{2}\, i\,, \qquad r = |\, z\,| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2 + 2} = 2$$

$$\sin \varphi_1 = \frac{y}{r} = \frac{\sqrt{2}}{2}, \qquad \varphi_1 = 45^\circ = \frac{\pi}{4}, \qquad \varphi_1 = \varphi \quad (x > 0, \quad y > 0)$$

$$\sqrt{2} + \sqrt{2}\, i = 2\, e^{\, i \frac{\pi}{4}} = 2 \left[\cos\left(\frac{\pi}{4} \right) + i \, \sin\left(\frac{\pi}{4} \right) \right]$$

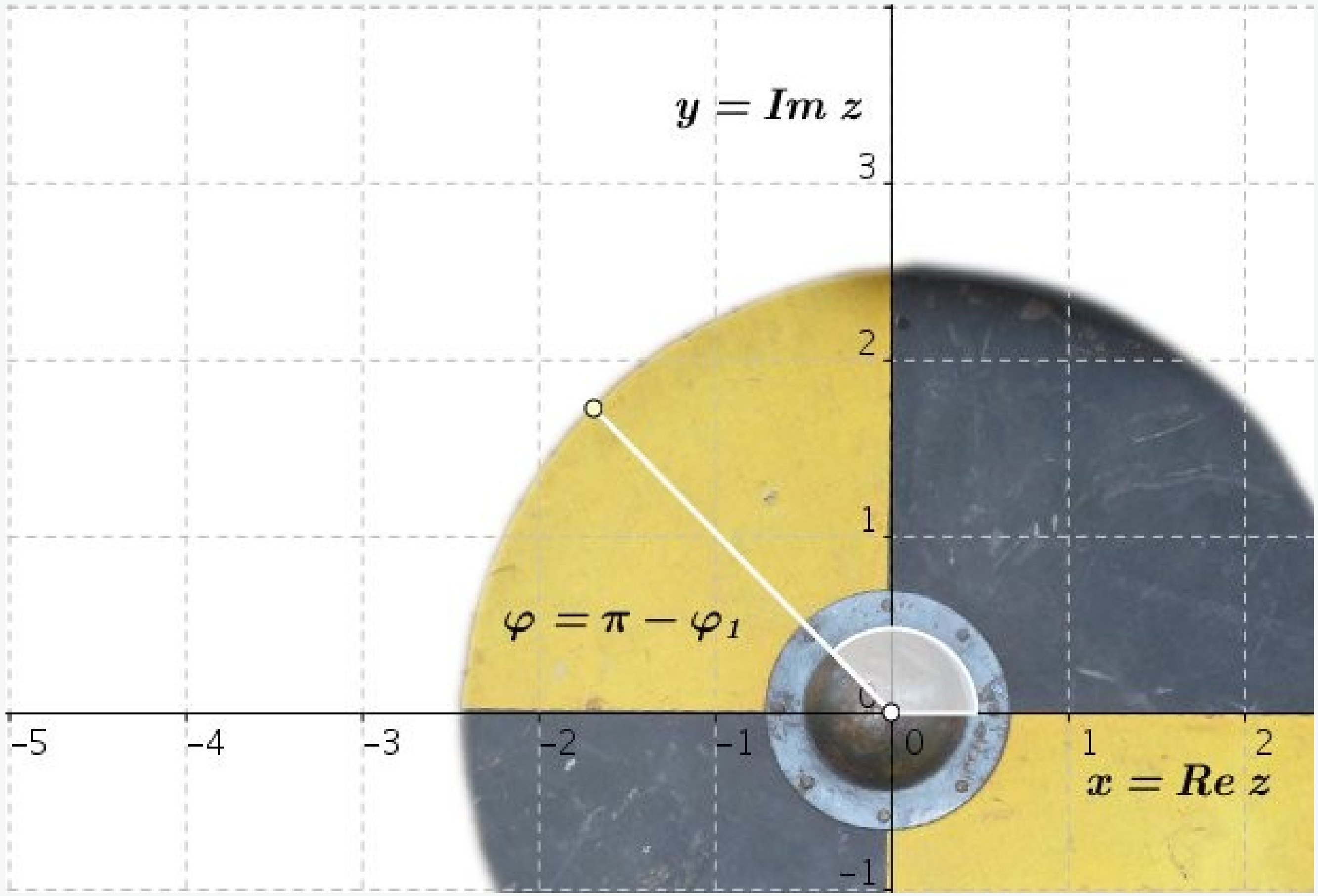

Abb. 5-2: Zur Bestimmung des Polarwinkels im zweiten Quadranten

$$z = -3 + 5\,i\,, \qquad r = |\,z\,| = \sqrt{(-3)^2 + 5^2} = \sqrt{34} \simeq 5.831$$

$$\sin\,\varphi_1 = \frac{|\,y\,|}{r} = \frac{5}{\sqrt{34}} \simeq 0.858\,, \qquad \varphi_1 = 59.04$$

$$x < 0,\ \ y > 0 \quad \Rightarrow \quad \varphi = \pi - \varphi_1 = 180^\circ - 59.04 \simeq 120.96^\circ$$

$$z = 5.83\ e^{i\,120.96^\circ} = 5.83\ (\cos\,(120.96^\circ) + i\,\sin\,(120.96^\circ))$$

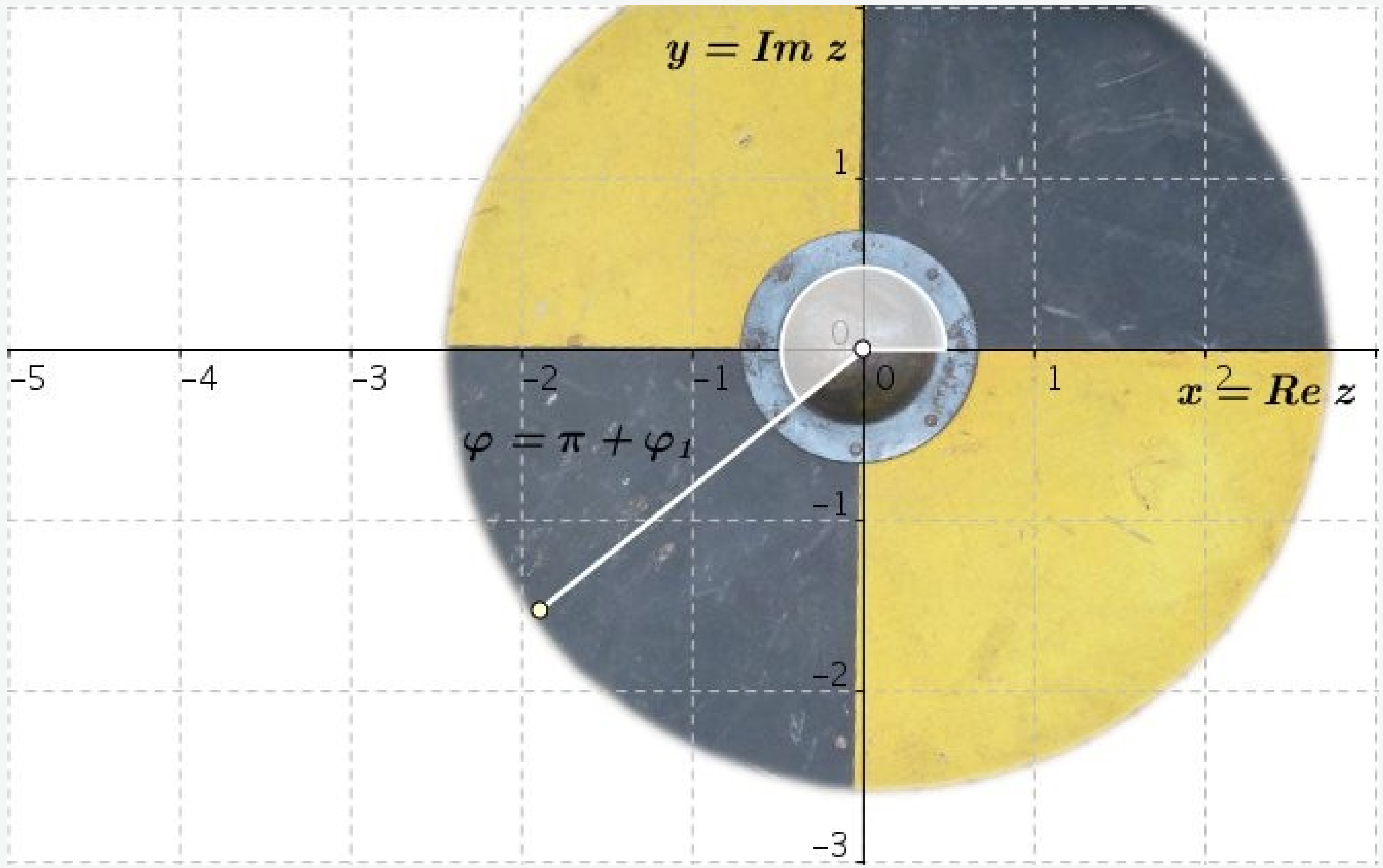

Abb. 5-3a: Zur Bestimmung des Polarwinkels im dritten Quadranten

$$z = -\frac{3\sqrt{3}}{2} - \frac{3}{2}\,i\,, \qquad r = |\,z\,| = \sqrt{\left(-\frac{3\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \frac{3}{2}\sqrt{3+1} = 3$$

$$\sin\,\varphi_1 = \frac{|\,y\,|}{r} = \frac{1}{2}, \quad \varphi_1 = 30^\circ = \frac{\pi}{6}, \quad \varphi = \pi + \varphi_1 = \frac{7\pi}{6} \quad (x < 0,\ \ y < 0)$$

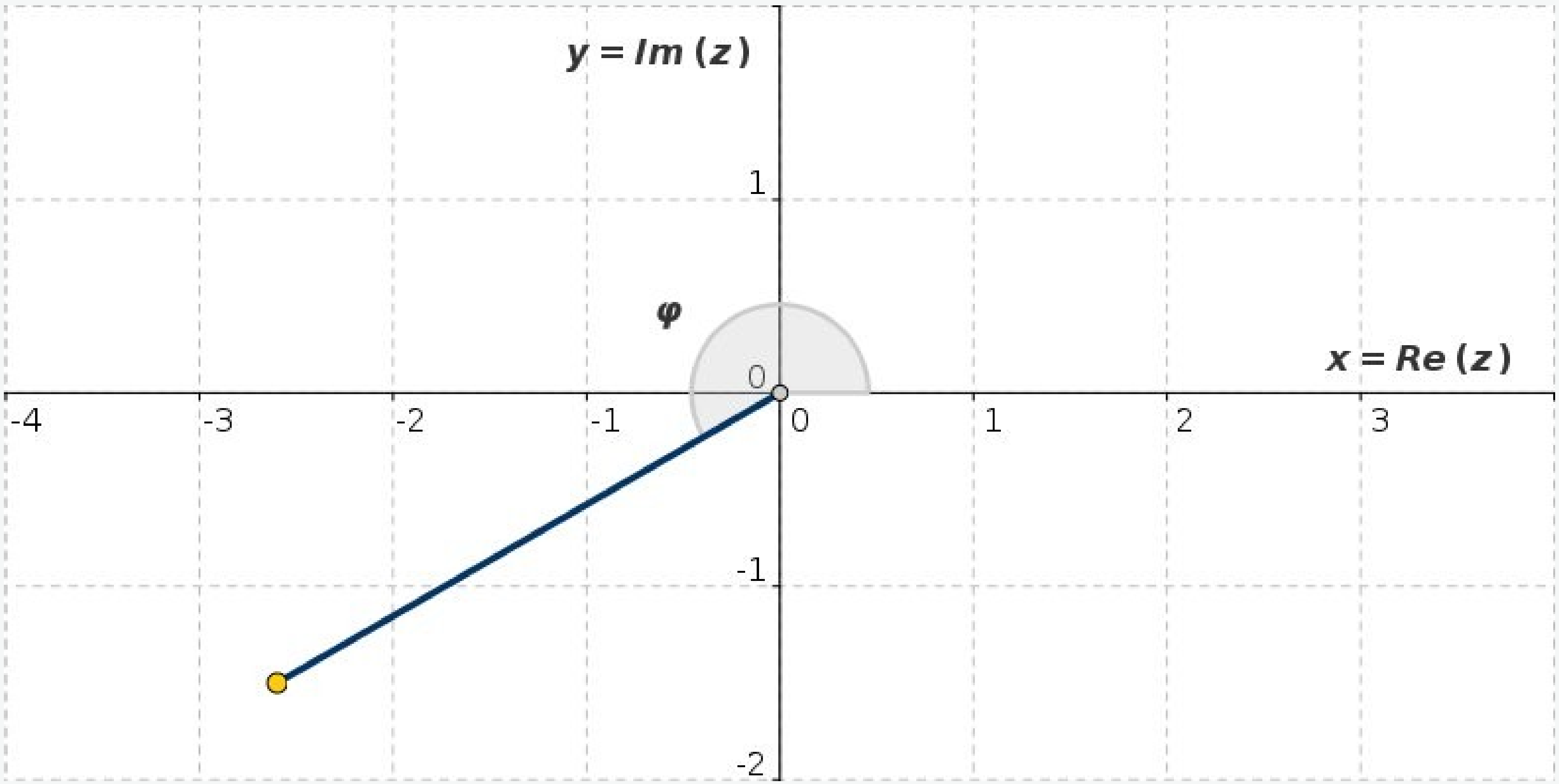

Abb. 5-3b: Graphische Darstellung der komplexen Zahl in der Gaußschen Zahlenebene

$$-\frac{3\sqrt{3}}{2} - \frac{3}{2}\, i = 3\, e^{i\,\frac{7\pi}{6}} = 3\, e^{i\pi}\, e^{i\,\frac{\pi}{6}} = -3\left[\cos\left(\frac{\pi}{6}\right) + i\,\sin\left(\frac{\pi}{6}\right)\right]$$

$$e^{i\pi} = \cos\pi + i\,\sin\pi = -1$$

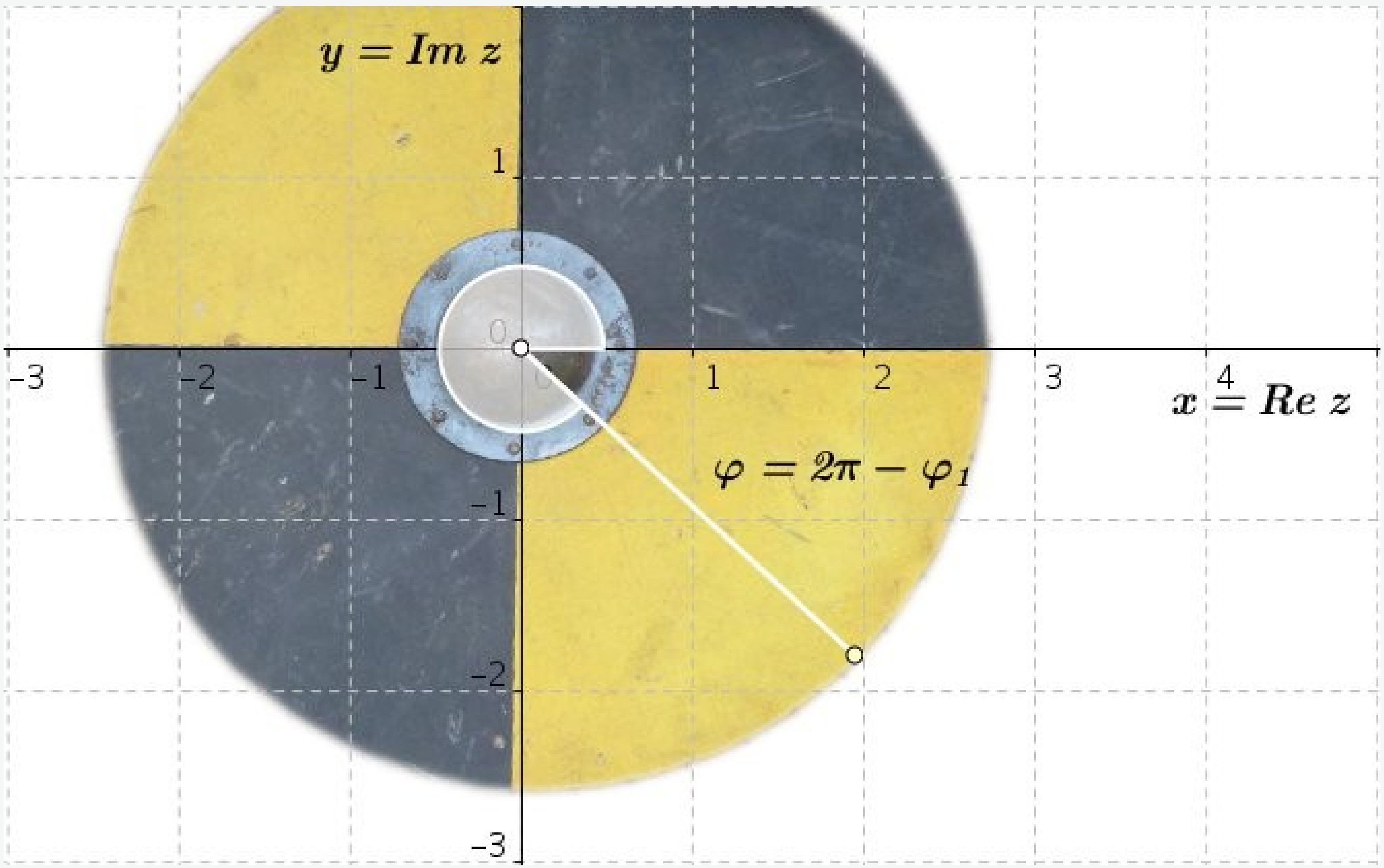

Abb. 5-4a: Zur Bestimmung des Polarwinkels im vierten Quadranten

$$z = 1 - i\,, \qquad r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\sin\varphi_1 = \frac{|y|}{r} = \frac{1}{\sqrt{2}}\,, \qquad \varphi_1 = 45^\circ = \frac{\pi}{4}\,, \qquad \varphi = 2\pi - \varphi_1 \quad (x > 0, \quad y < 0)$$

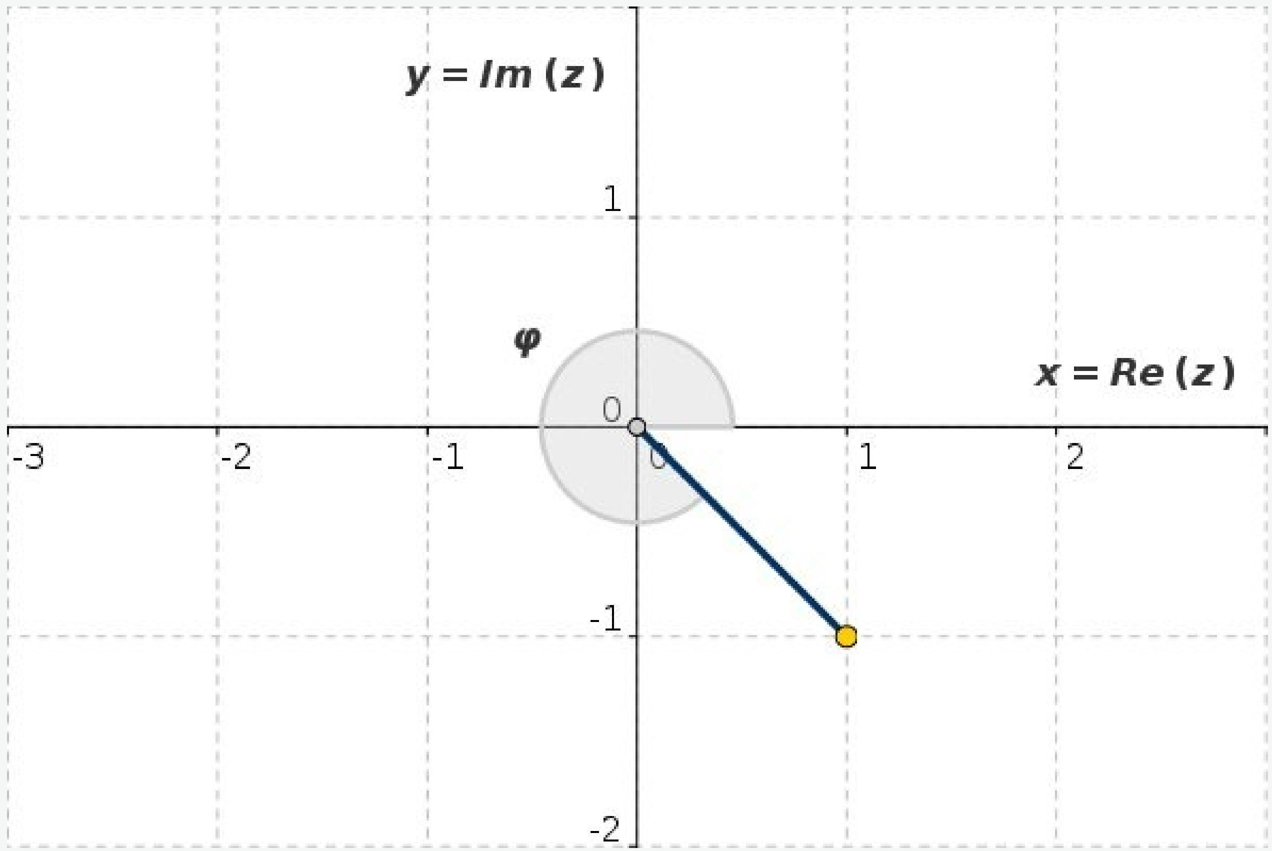

Abb. 5-4b: Graphische Darstellung der komplexen Zahl in der Gaußschen Zahlenebene

$$1 - i = \sqrt{2}\ e^{\,i\,\left(2\pi - \frac{\pi}{4}\right)} = \sqrt{2}\ e^{2\pi i}\ e^{-i\,\frac{\pi}{4}} = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) - i\ \sin\left(\frac{\pi}{4}\right)\right]$$

$$e^{2\pi i} = \cos(2\pi) + i\ \sin(2\pi) = 1$$

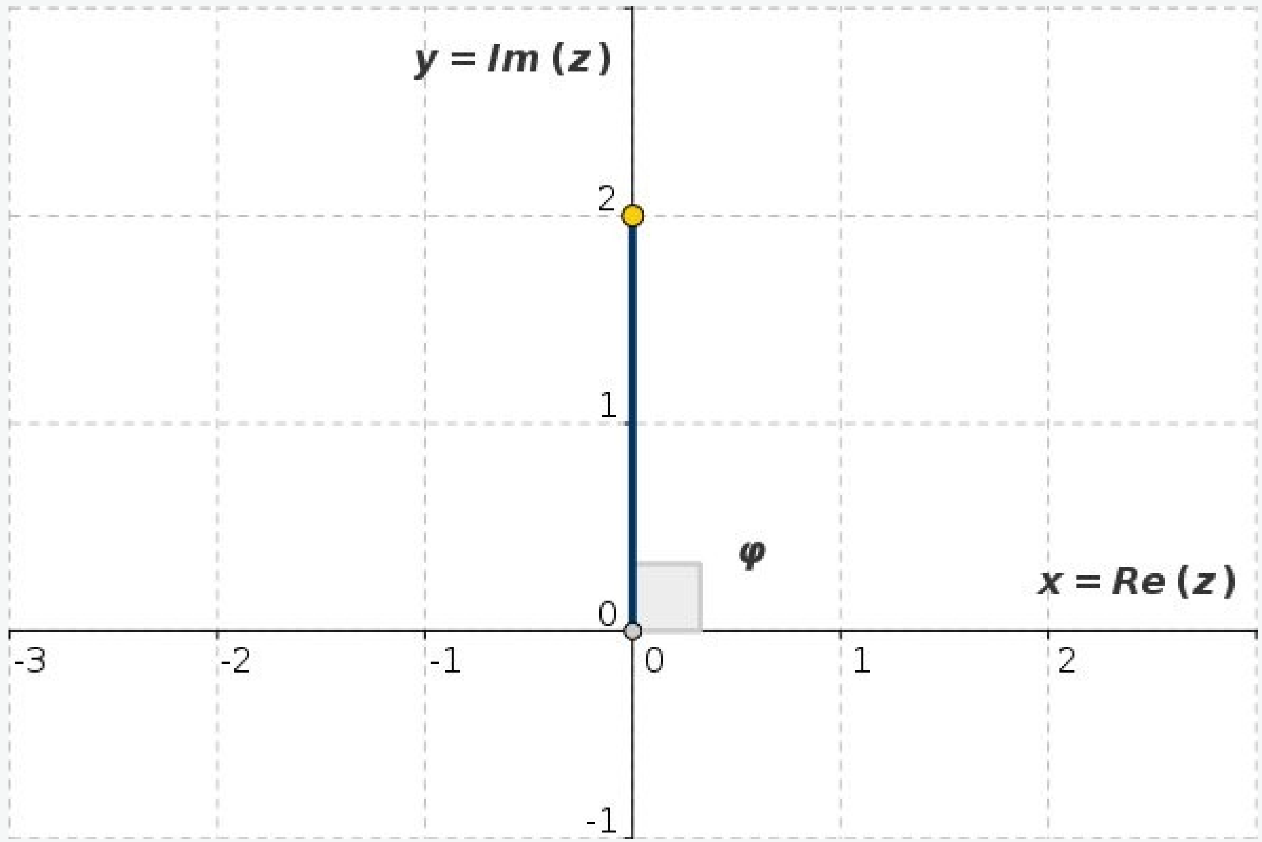

Abb. 5-5: Graphische Darstellung der komplexen Zahl in der Gaußschen Zahlenebene

$$z = 2\,i\,, \qquad r = |\,z\,| = \sqrt{0^2 + 2^2} = 2$$

$$\sin\varphi = \frac{2}{2} = 1, \quad x = 0, \quad y > 0, \quad \varphi = \frac{\pi}{2} = 90^\circ, \quad z = 2\,e^{i\,90^\circ}$$

Die in kartesischenr Form gegebenen komplexen Zahlen sind in Polarform umzurechnen

Aufgabe 6: $z = -5 + 4\,i$

Aufgabe 7: $z = 2 + \sqrt{7}\; i$

Aufgabe 8: $z = -\sqrt{3} - \sqrt{8}\; i$

Aufgabe 9: $z = \sqrt{7} - i$

Lösung 6: $z = -5 + 4\,i\,, \qquad r = |\,z\,| = \sqrt{41}\,, \quad \varphi \simeq 141.34^\circ$

Lösung 7: $z = 2 + \sqrt{7}\,i\,, \qquad r = |\,z\,| = \sqrt{11}\,, \quad \varphi \simeq 52.91^\circ$

Lösung 8: $z = -\sqrt{3} - \sqrt{8}\,i\,, \qquad r = |\,z\,| = \sqrt{11}\,, \quad \varphi \simeq -121.48^\circ$

Lösung 9: $z = \sqrt{7} - i\,, \qquad r = |\,z\,| = 2\sqrt{2}\,, \quad \varphi \simeq -20.71^\circ$