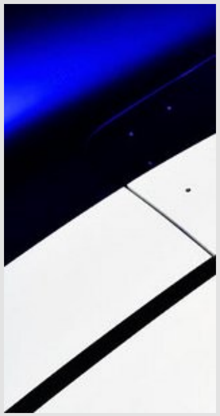
http://www.flickr.com/photos/dwhuntley/2890084872/

Gebrochenrationale Funktionen



$$f(x) = \frac{Z(x)}{N(x)} = \frac{a_m x^m + a_{m-1} x^{m-1} + \ldots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \ldots + b_1 x + b_0}$$

$Z(x)$, $N(x)$ – ganzrationale Funktionen (Polynomfunktionen)

$n > m$ – echt gebrochenrationale Funktion

$n \leq m$ – unecht gebrochenrationale Funktion

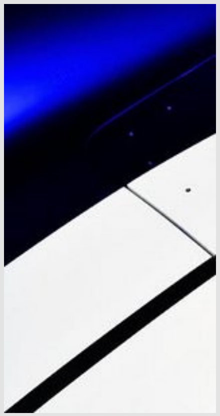
Jede unecht gebrochenrationale Funktion kann in dieser Form dargestellt werden:

$$f(x) = \frac{Z(x)}{N(x)} = P_{m-n}(x) + \frac{\tilde{Z}(x)}{\tilde{N}(x)}$$

$\dfrac{\tilde{Z}(x)}{\tilde{N}(x)}$ – eine echt gebrochenrationale Funktion

$P_{m-n}(x)$ – Polynomfunktion $(m-n)$. Grades

$P_0(x)$ – eine Konstante



$$x \to \pm\infty$$

$n > m$ – $f(x)$ ist eine echt gebrochenrationale Funktion

$$\lim_{x \to \pm\infty} f(x) = 0$$

Beispiele:

$$\lim_{x \to \pm\infty} \frac{1}{x} = 0$$

$$\lim_{x \to \pm\infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to \pm\infty} \frac{8x}{x^3 + 2x} = 0$$

Der Graph nähert sich unbegrenzt der x-Achse. Diese Achse nennt man dann eine Asymptote der Kurve mit der Asymptotengleichung

$$y_A = 0$$

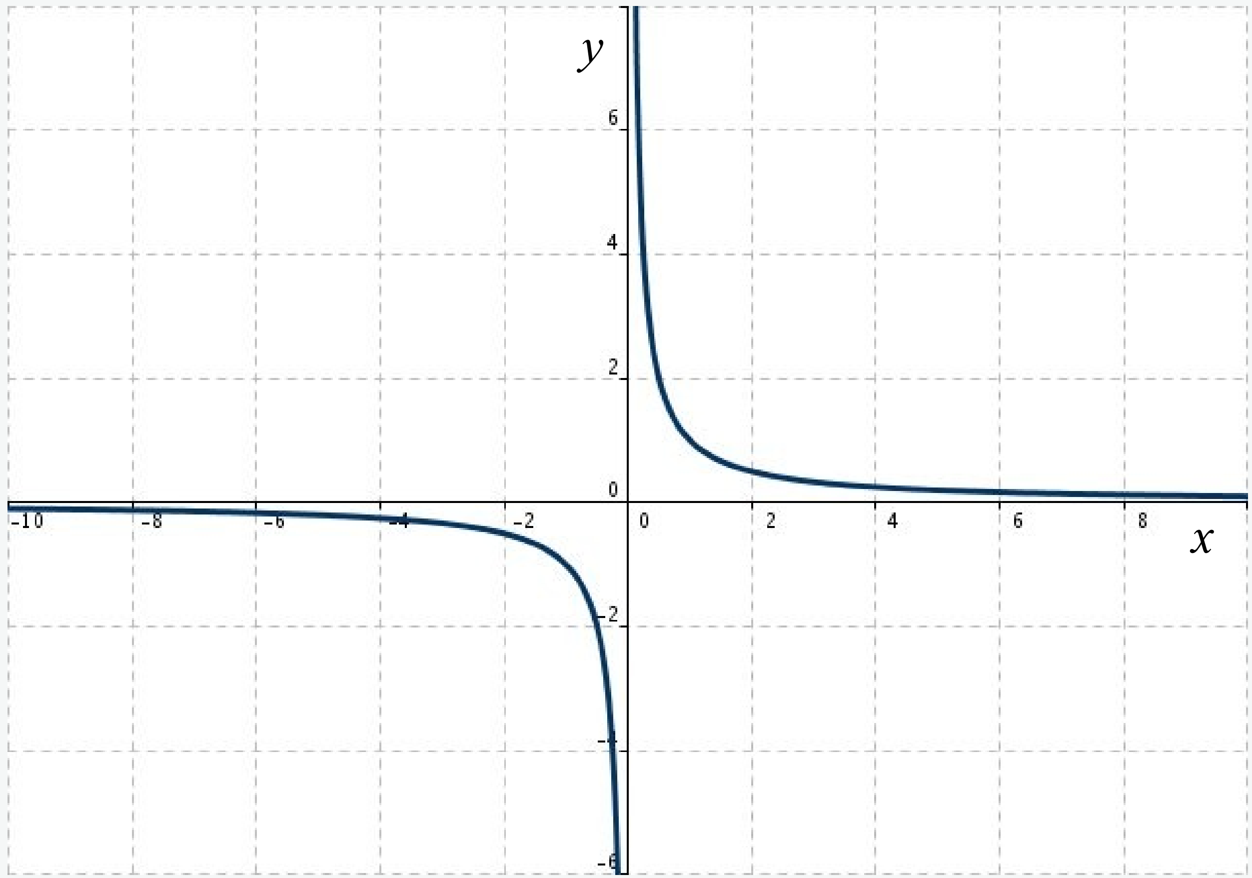


Abb. 2-1: Echt gebrochenrationale Funktion y = f (x)

$$f(x) = \frac{1}{x}$$

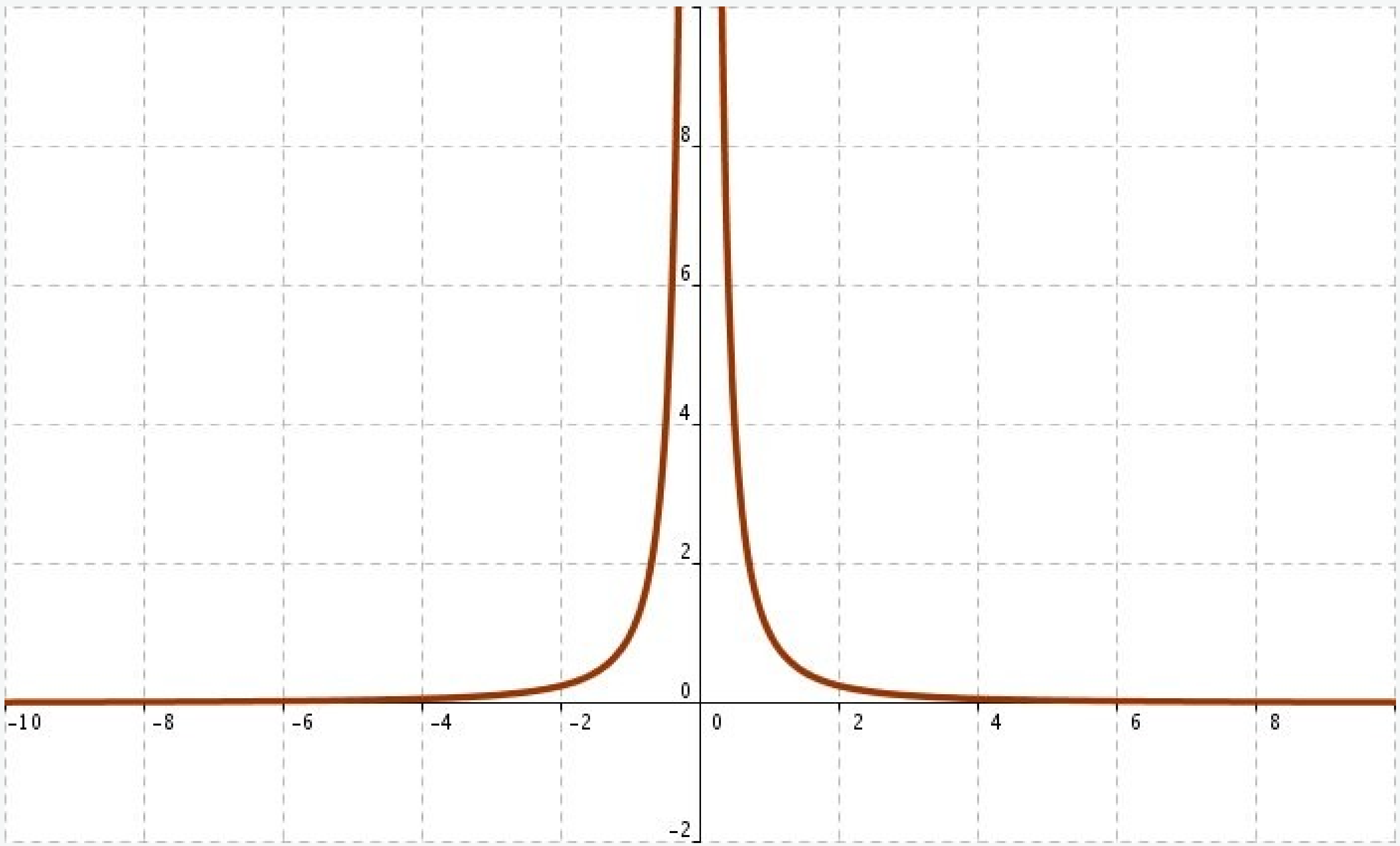


Abb. 2-2: Echt gebrochenrationale Funktion $y = f(x)$

$$f(x) = \frac{1}{x^2}$$

Echt gebrochenrationale Funktion: Beispiel 3

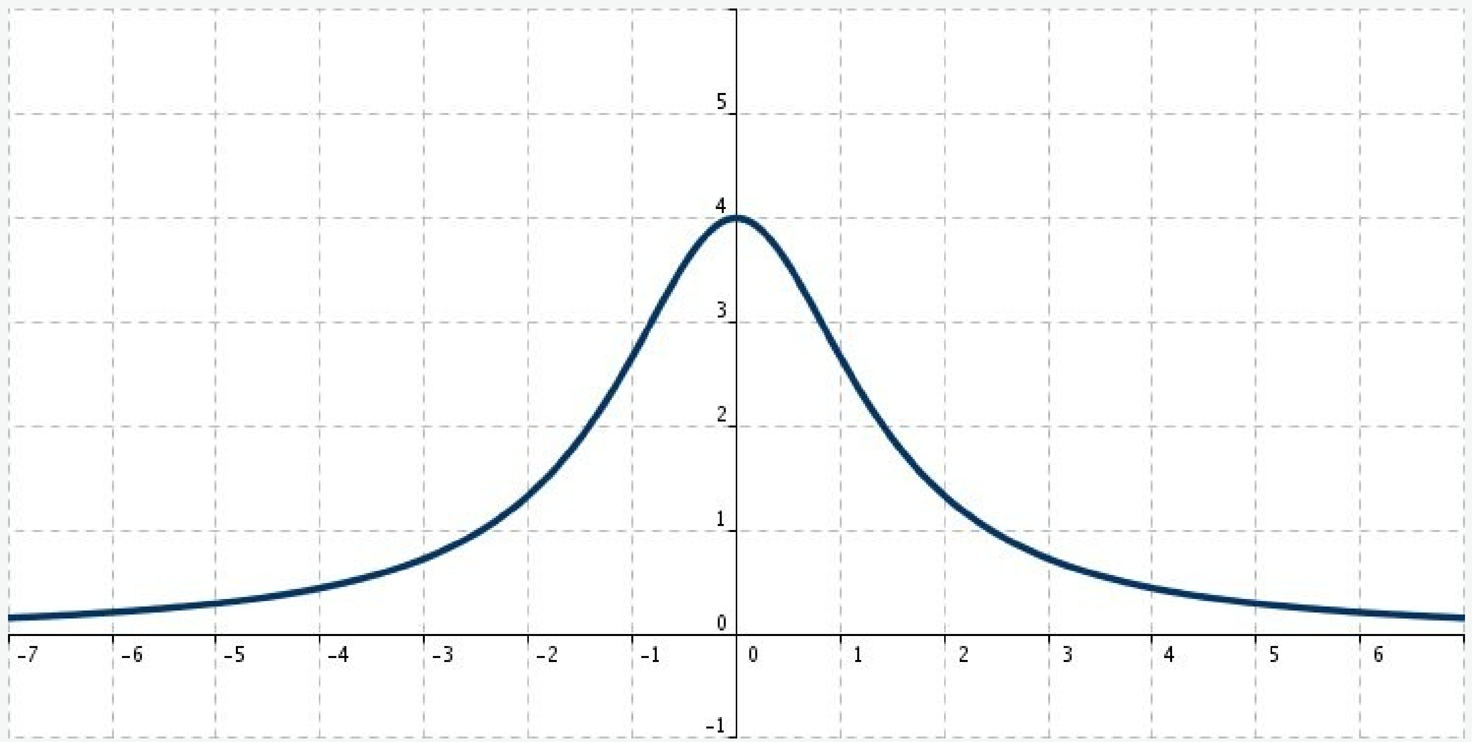


Abb. 2-3: Echt gebrochenrationale Funktion y = f (x)

$$f(x) = \frac{8x}{x^3 + 2x}$$

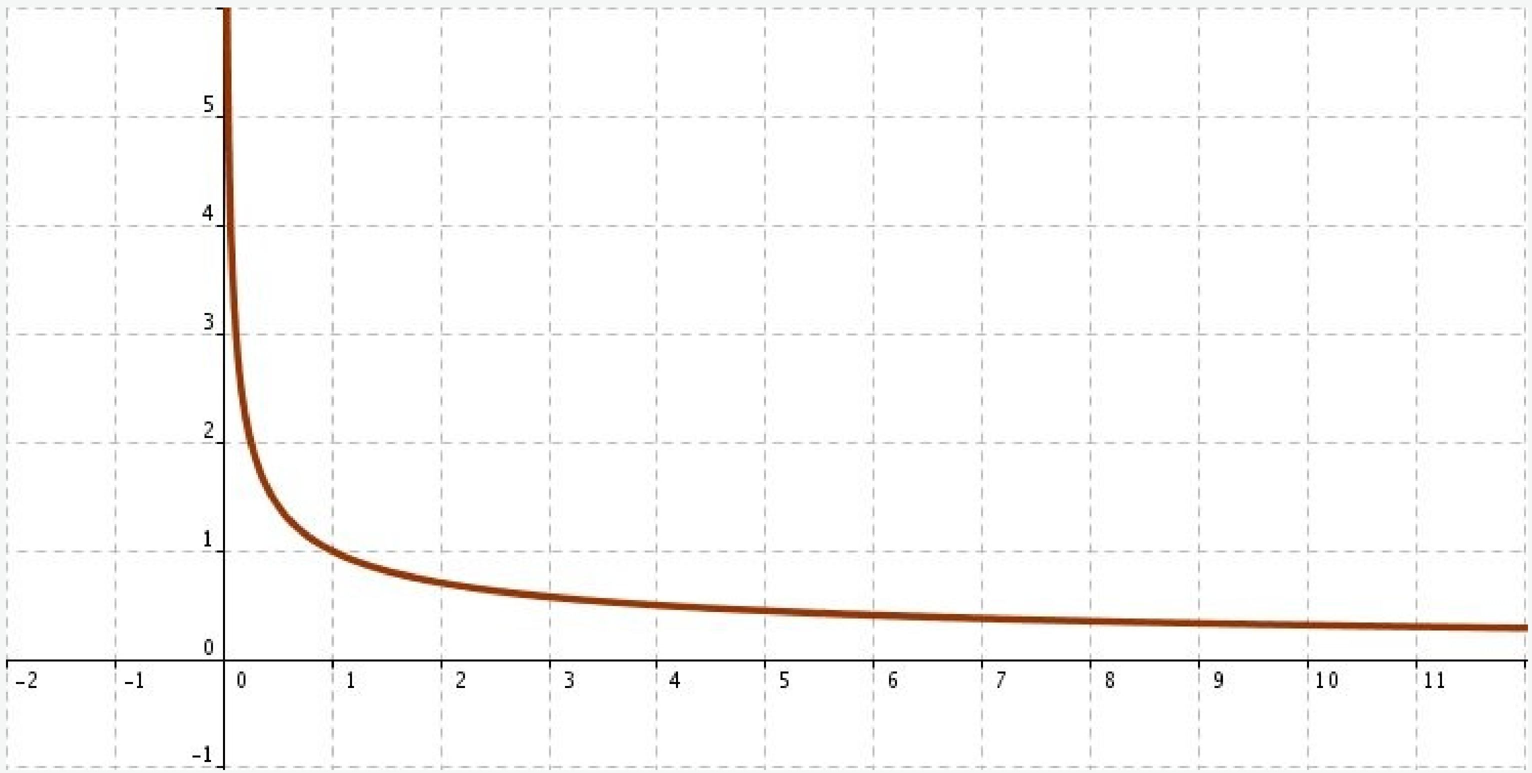
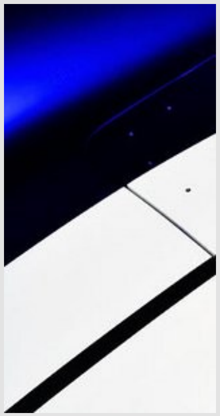


Abb. 2-4: Echt gebrochenrationale Funktion y = f (x)

$$f(x) = \frac{1}{\sqrt{x}}$$



$$\boxed{x \to \pm \infty}$$

$$m = n$$

$$f(x) = \frac{Z(x)}{N(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \ldots + b_1 x + b_0}$$

$$\lim_{x \to \pm \infty} f(x) = \frac{a_n}{b_n} \quad \Rightarrow \quad y_A = \frac{a_n}{b_n}$$

Beispiele:

$$\lim_{x \to \pm \infty} \frac{2x}{x + 3} = 2$$

$$\lim_{x \to \pm \infty} \frac{-3x^2 + 7x - 2}{2x^2 - 5} = -\frac{3}{2}$$

$$\lim_{x \to \pm \infty} \frac{x^3}{3x^3 - 2x^2} = \frac{1}{3}$$

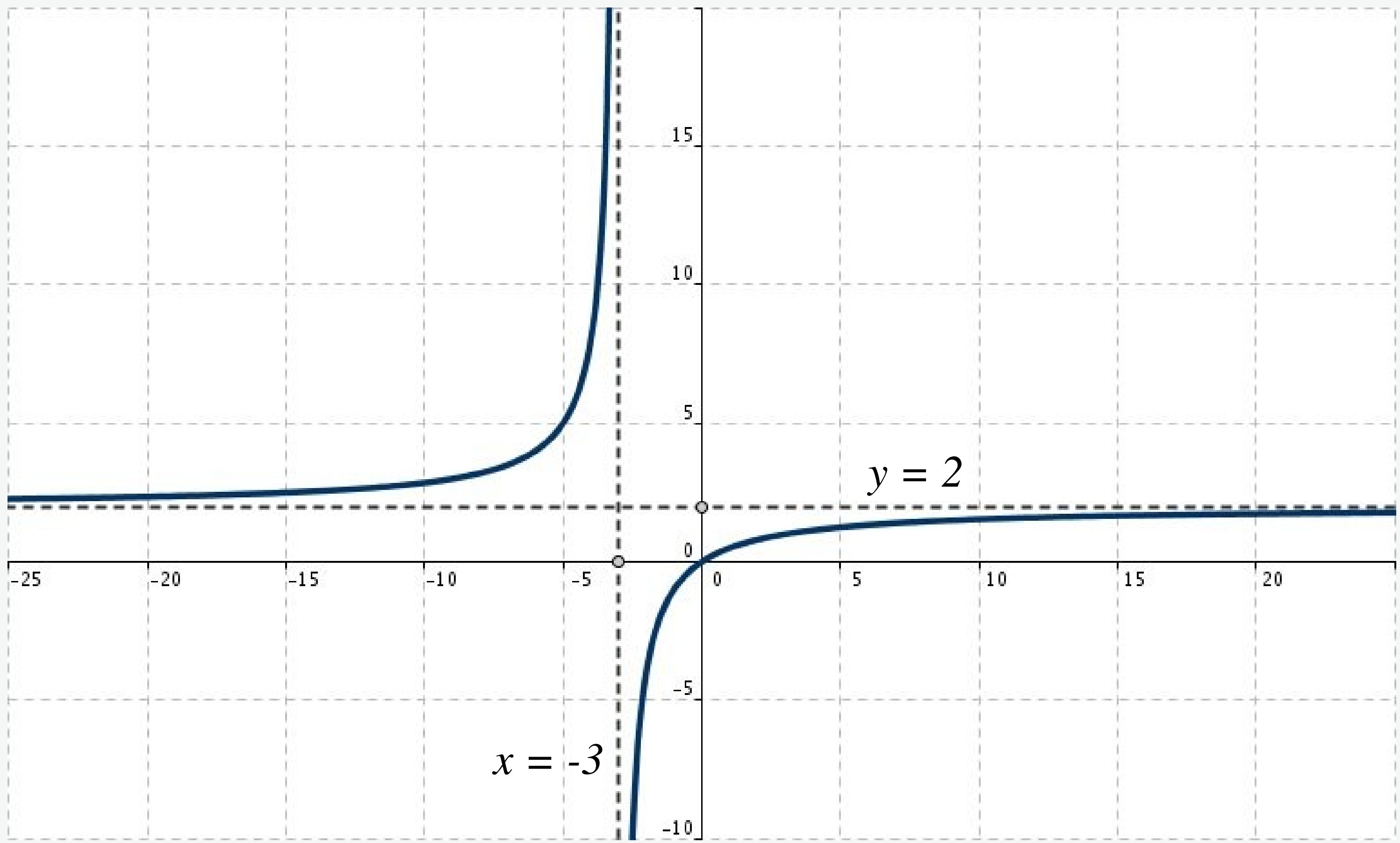


Abb. 3-1: Unecht gebrochenrationale Funktion $y = f(x)$

$$f(x) = \frac{2x}{x + 3}$$

$y = 2$ – waagerechte Asymptote, $x = -3$ – senkrechte Asymptote

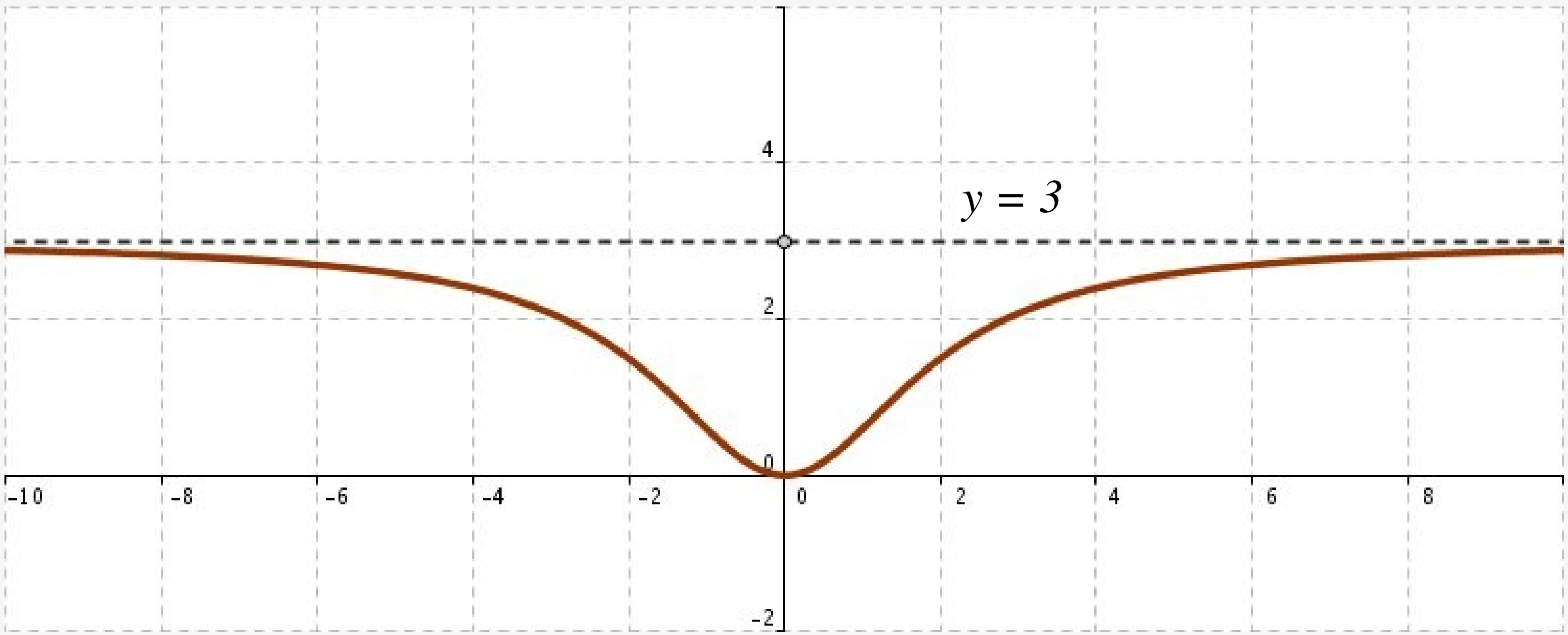


Abb. 3-2: Unecht gebrochenrationale Funktion $y = f(x)$

$$f(x) = \frac{3x^2}{x^2 + 4}$$

$y = 3$ – waagerechte Asymptote

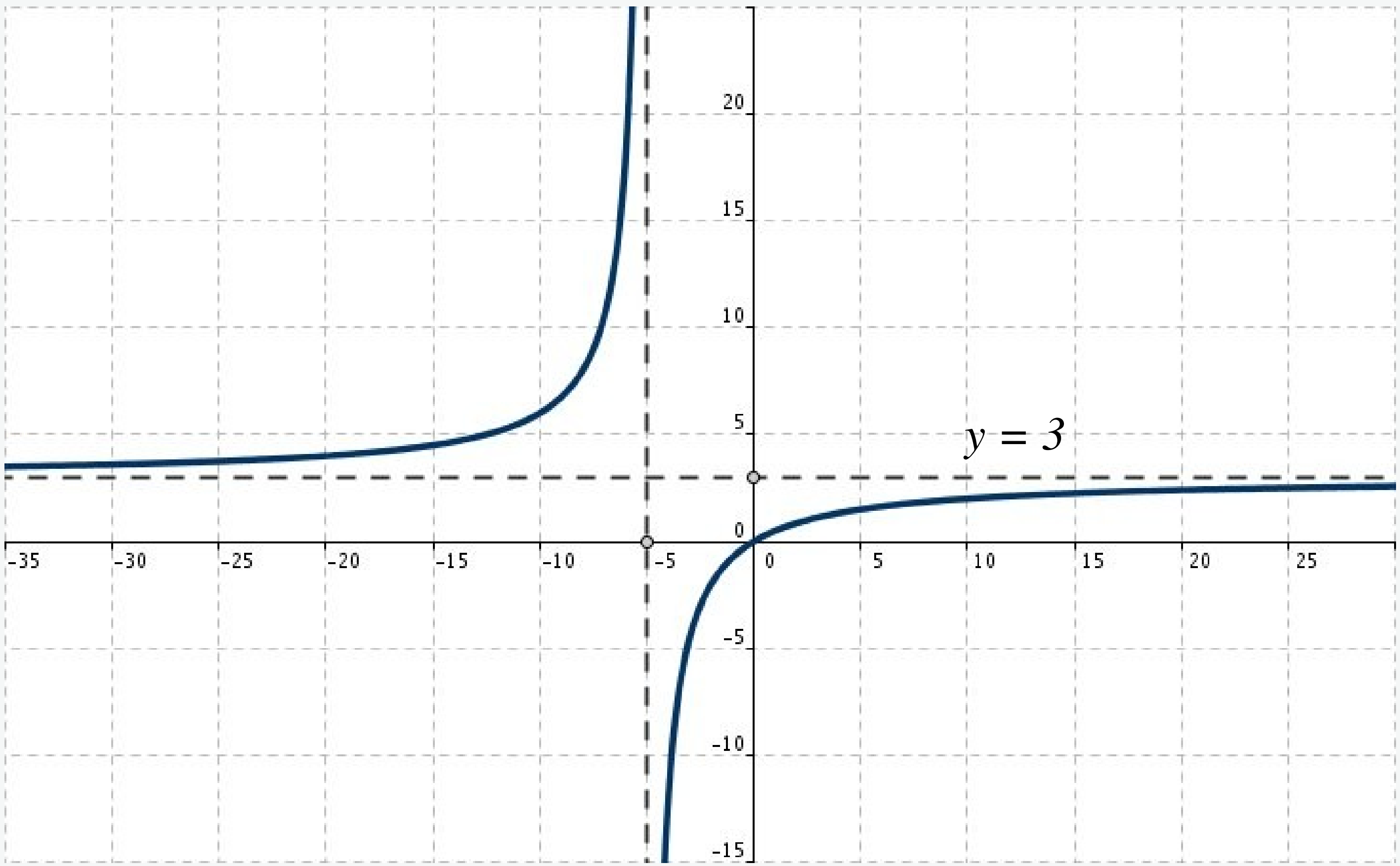


Abb. 3-3: Unecht gebrochenrationale Funktion y = f (x)

$$f(x) = \frac{3x^2}{x^2 + 5x}$$

$y = 3$ – waagerechte Asymptote, $x = -5$ – senkrechte Asymptote

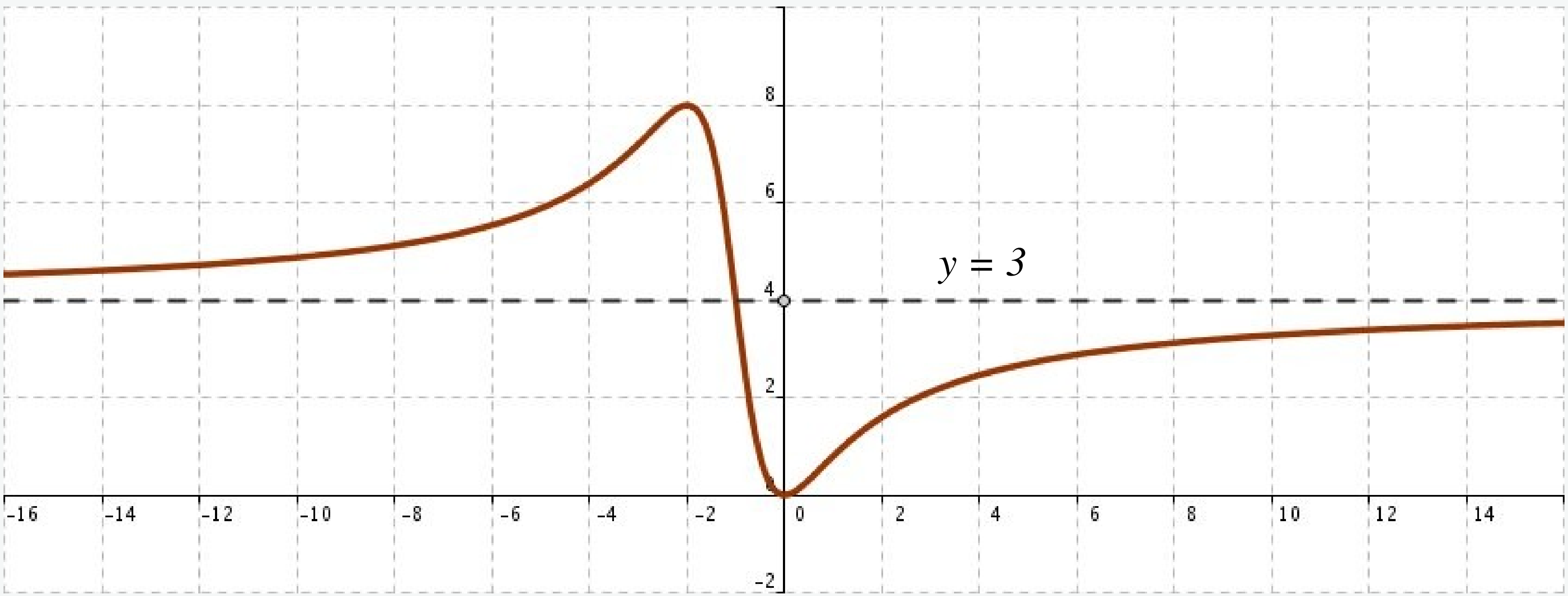


Abb. 3-4: Unecht gebrochenrationale Funktion $y = f(x)$

$$f(x) = \frac{4x^4}{x^4 + 2x^3 + 2x^2}$$

$y = 4$ – waagerechte Asymptote