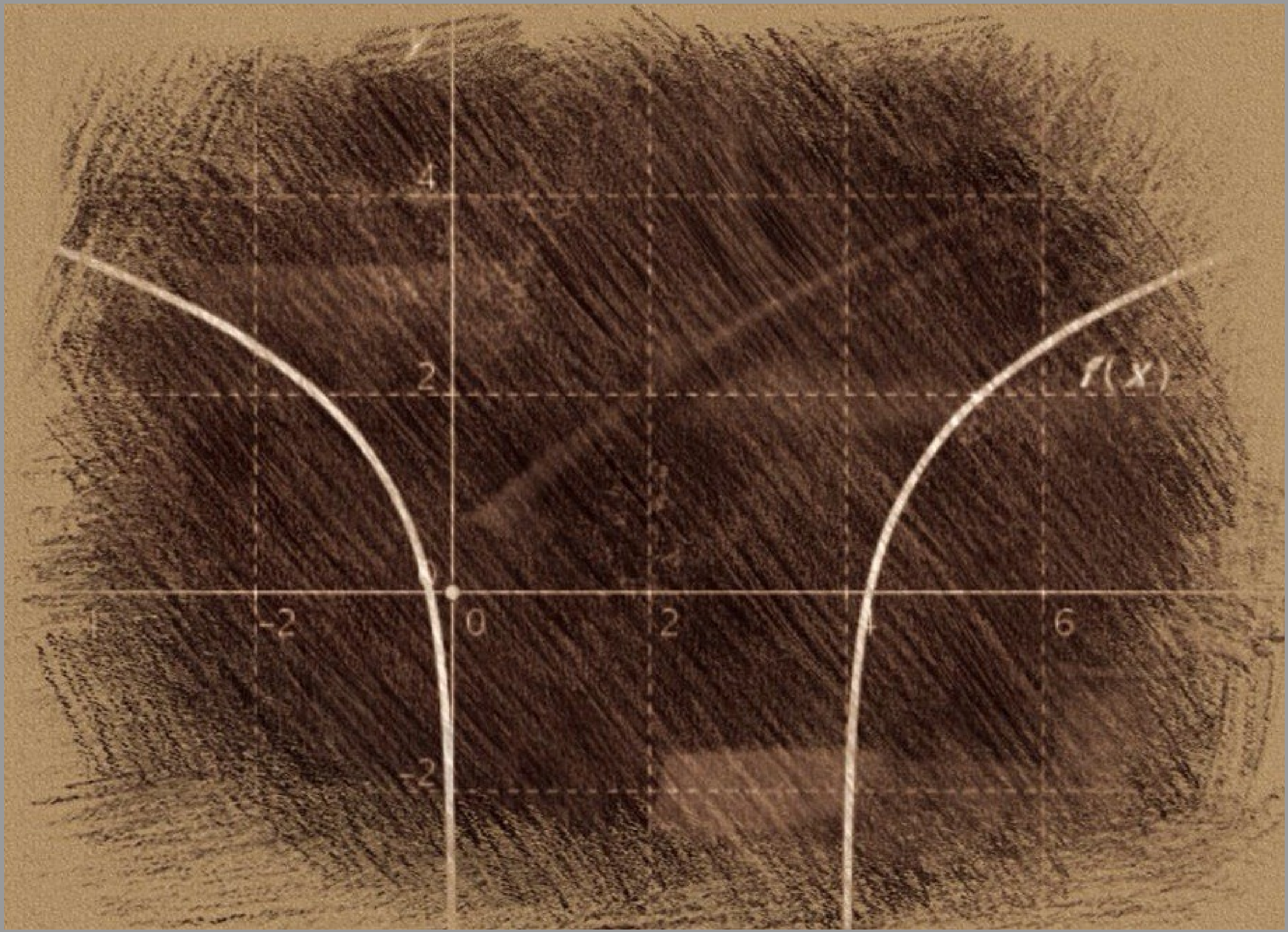


Ableitung logaritmischer Funktionen: Aufgaben

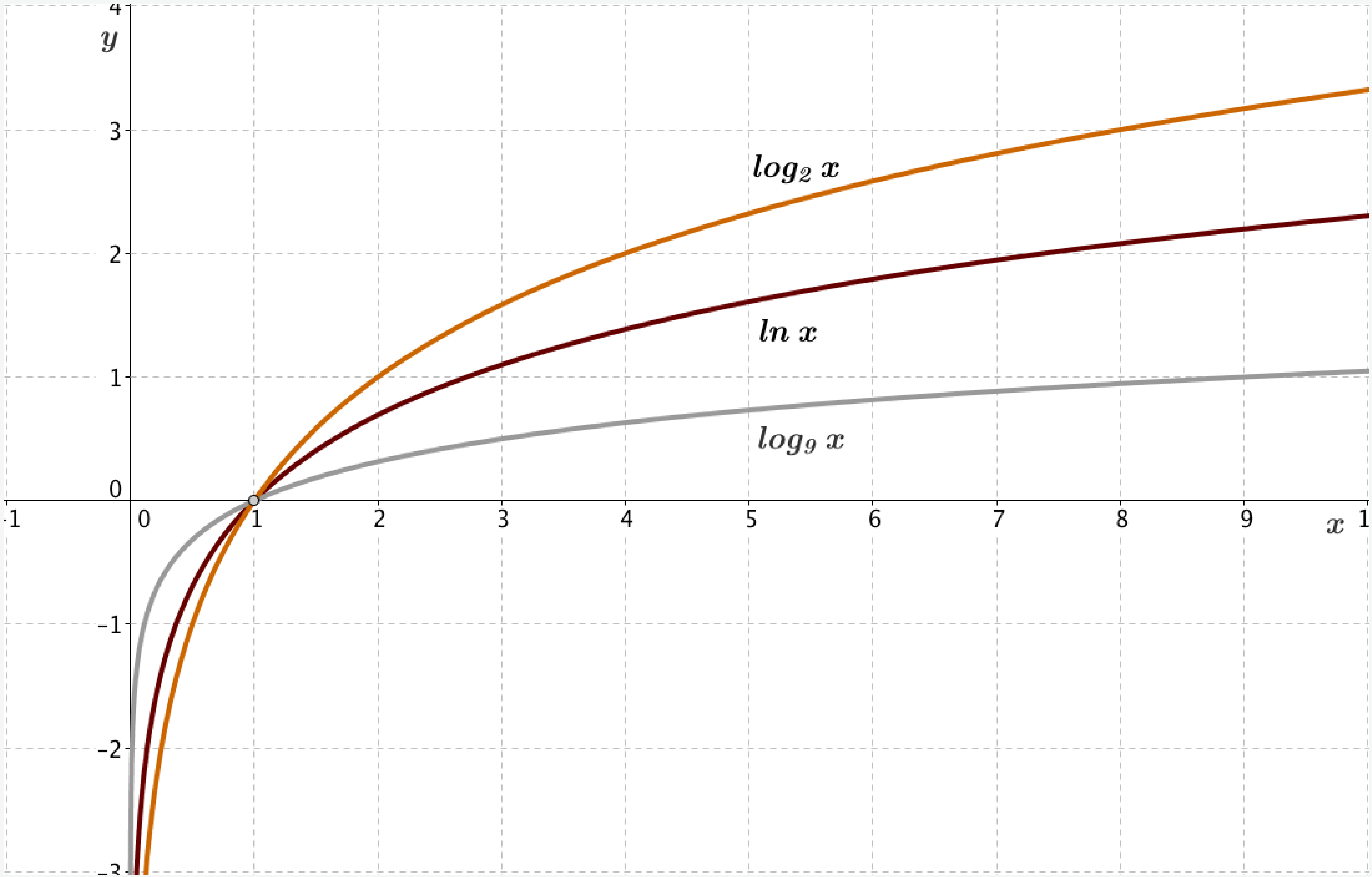
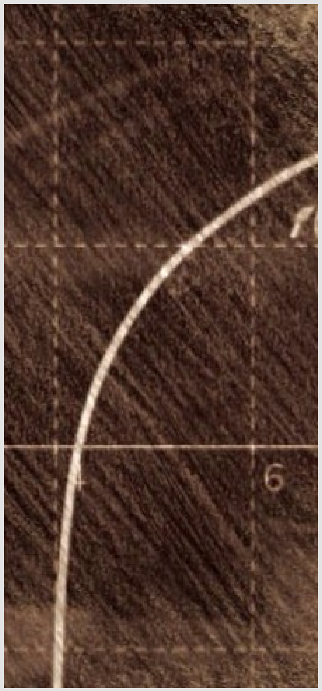


Abb. 1: Logarithmusfunktionen zur Basis 2, e und 9

Logarithmengesetze



$$\log_b (x \cdot y) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b (x^n) = n \log_b x$$

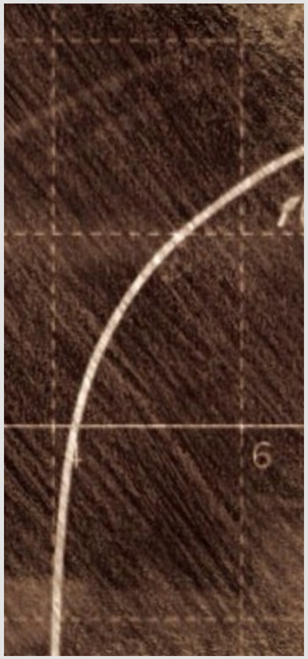
Sonderfälle:

$$\log_b \sqrt[n]{x} = \frac{1}{n} \log_b x$$

$$\log_b \left(\frac{1}{x} \right) = -\log_b x$$

$$\log_b b^x = x, \qquad e^{\ln x} = x$$

$$\log_b e = 1, \qquad \log_b 1 = 0$$



Bestimmen Sie die erste Ableitung folgender Funktionen:

Aufgabe 1:

$$a)\quad f(x) = \ln\left(\sqrt{x}\right), \qquad g(x) = \ln\left(\sqrt[4]{x}\right), \qquad h(x) = \ln\left(\frac{1}{x}\right)$$

$$b)\quad f(x) = \ln(x-9), \qquad g(x) = \ln\left(\sqrt{x-9}\right)$$

$$h(x) = \ln\left(\sqrt{x} - 9\right)$$

Aufgabe 2:

$$a)\quad f(x) = \ln\sqrt{x^2+2}\,, \qquad g(x) = \ln\frac{1}{\sqrt{x^2+2}}$$

$$h(x) = \ln\left(1 - \sqrt{x^2+2}\,\right)$$

$$b)\quad f(x) = \ln\sqrt{\frac{x}{x-3}}\,, \qquad g(x) = \ln\frac{x+2}{\sqrt{x^2-4}}$$

Lösung 1:

$$a)\ f'(x) = \frac{1}{2x}, \qquad g' = \frac{1}{4x}, \qquad h(x) = -\frac{1}{4x}$$

$$b)\ f'(x) = \frac{1}{x-9}, \qquad g'(x) = \frac{1}{2(x-9)}$$

$$h'(x) = \frac{1}{2\sqrt{x}\,(\sqrt{x}-9)}$$

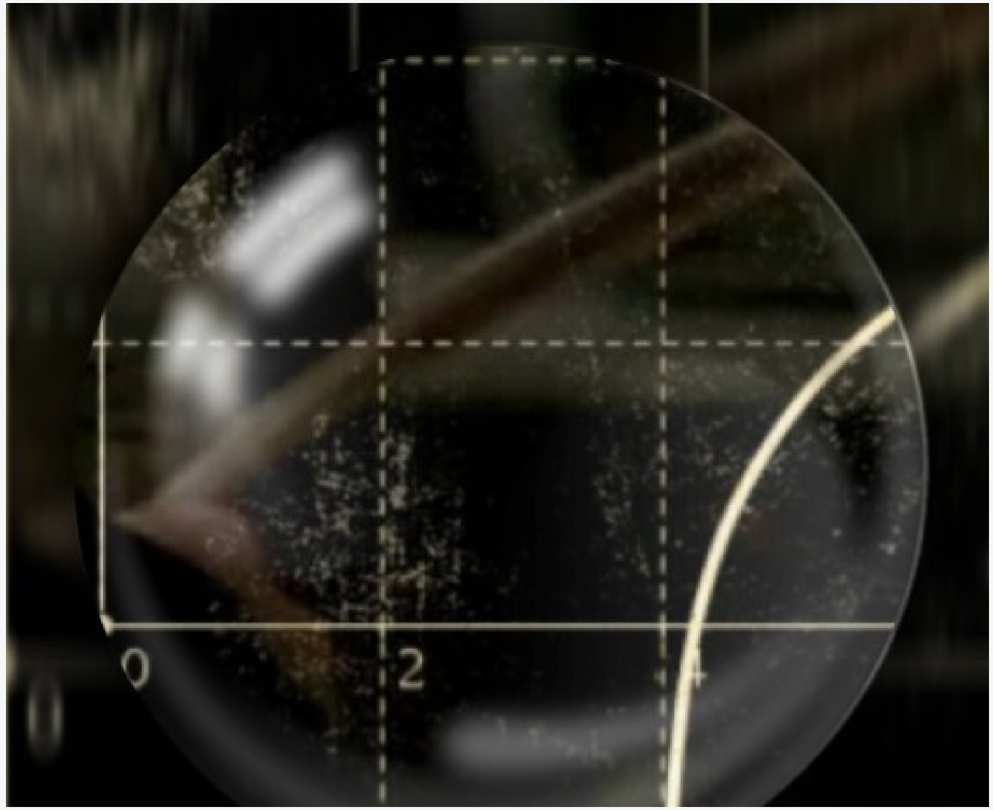
Lösung 2:

$$a)\ f'(x) = \left(\ln\sqrt{x^2+2}\right)' = \frac{x}{x^2+2}, \qquad g'(x) = -f'(x)$$

$$h'(x) = \left(\ln\left(1-\sqrt{x^2+2}\right)\right)' = \frac{x}{\sqrt{x^2+2}\,\left(\sqrt{x^2+2}-1\right)}$$

$$b)\ f'(x) = \left(\ln\sqrt{\frac{x}{x-3}}\right)' = -\frac{3}{2x(x-3)}$$

$$g'(x) = \left(\ln\frac{x+2}{\sqrt{x^2-4}}\right)' = -\frac{2}{x^2-4}$$



Bestimmen Sie die Definitionsbereiche der Funktionen *f (x)* und *g (x)* und die ersten Ableitungen:

$$f(x) = \ln(x^2 - 4x)$$

$$g(x) = \ln x + \ln(x - 4)$$

$$f(x) = \ln(x^2 - 4x), \qquad x^2 - 4x = x(x-4) > 0$$

$$x > 0, \quad x - 4 > 0 \quad \cup \quad x < 0, \quad x - 4 < 0$$

$$x > 0, \quad x > 4 \quad \cup \quad x < 0, \quad x < 4$$

$$D_f = (-\infty, 0) \cup (4, \infty)$$

$$g(x) = \ln x + \ln(x-4)$$

$$D(\ln x) = (0, \infty), \qquad D(\ln(x-4)) = (4, \infty)$$

$$D_g = (4, \infty), \qquad D_f \neq D_g$$

$$f'(x) = \left(\ln(x^2 - 4x)\right)' = \frac{2(x-2)}{x(x-4)}, \qquad x \in D_f$$

$$g'(x) = \left(\ln x + \ln(x-4)\right)' = \frac{2(x-2)}{x(x-4)}, \qquad x \in D_g$$

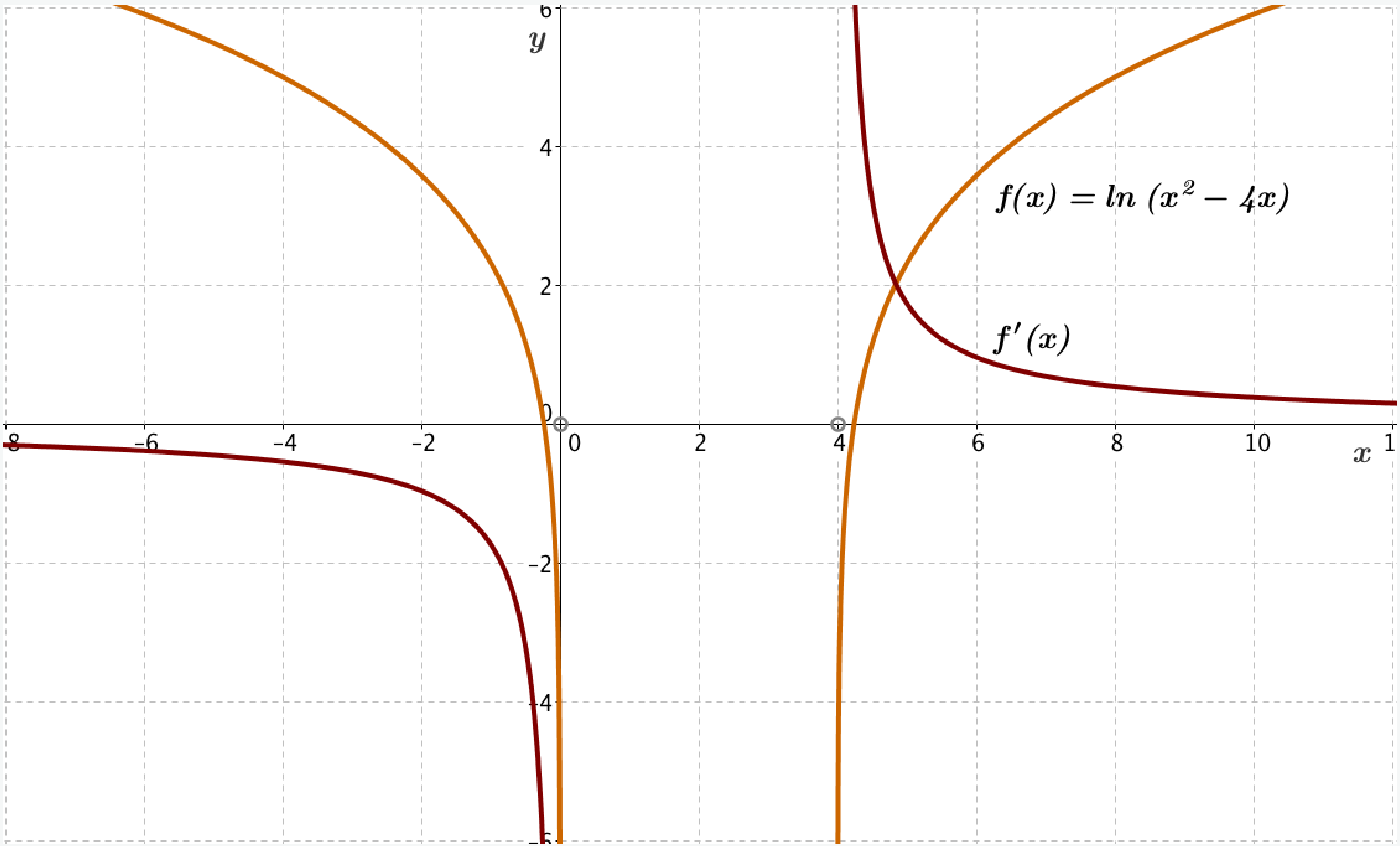


Abb. L3-1: Die Funktion f (x) und ihre Ableitung f' (x)

$$f\,(x) = \ln\,(x^2 - 4\,x)$$

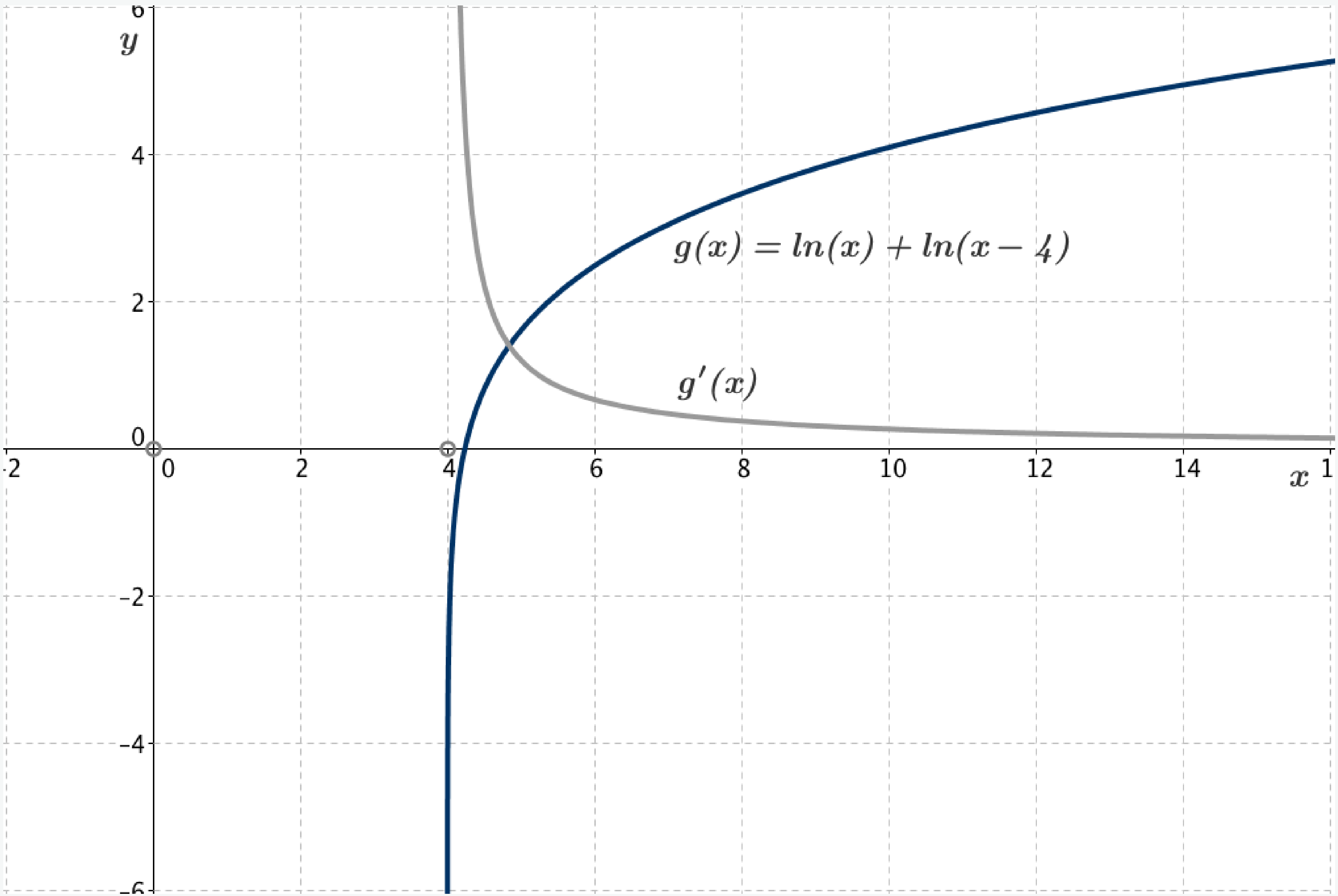
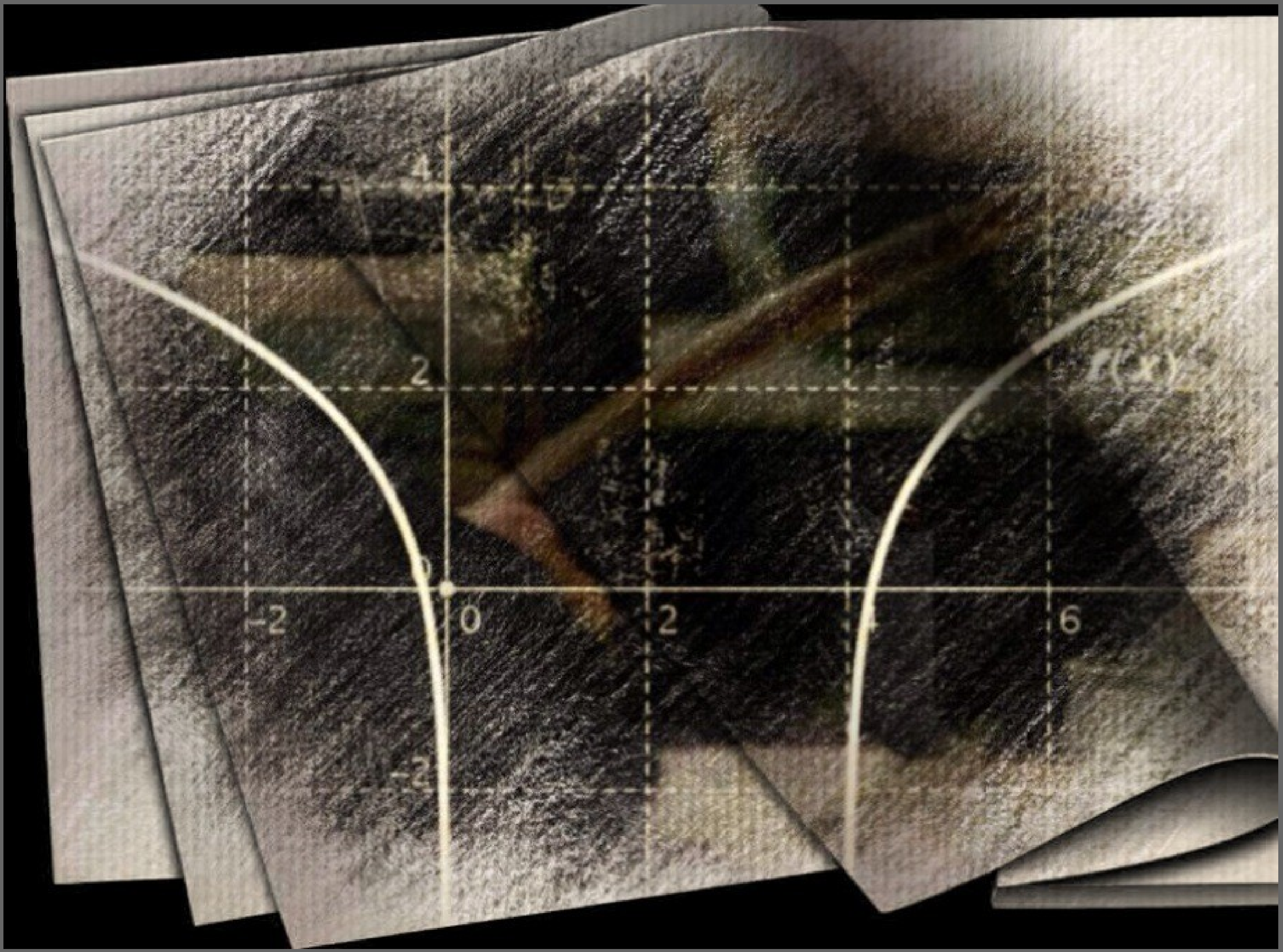


Abb. L3-2: Die Funktion g (x) und ihre Ableitung g' (x)

$$g(x) = \ln x + \ln(x - 4)$$


4
2
0
-2
-2
0
2
4
6
f(x)