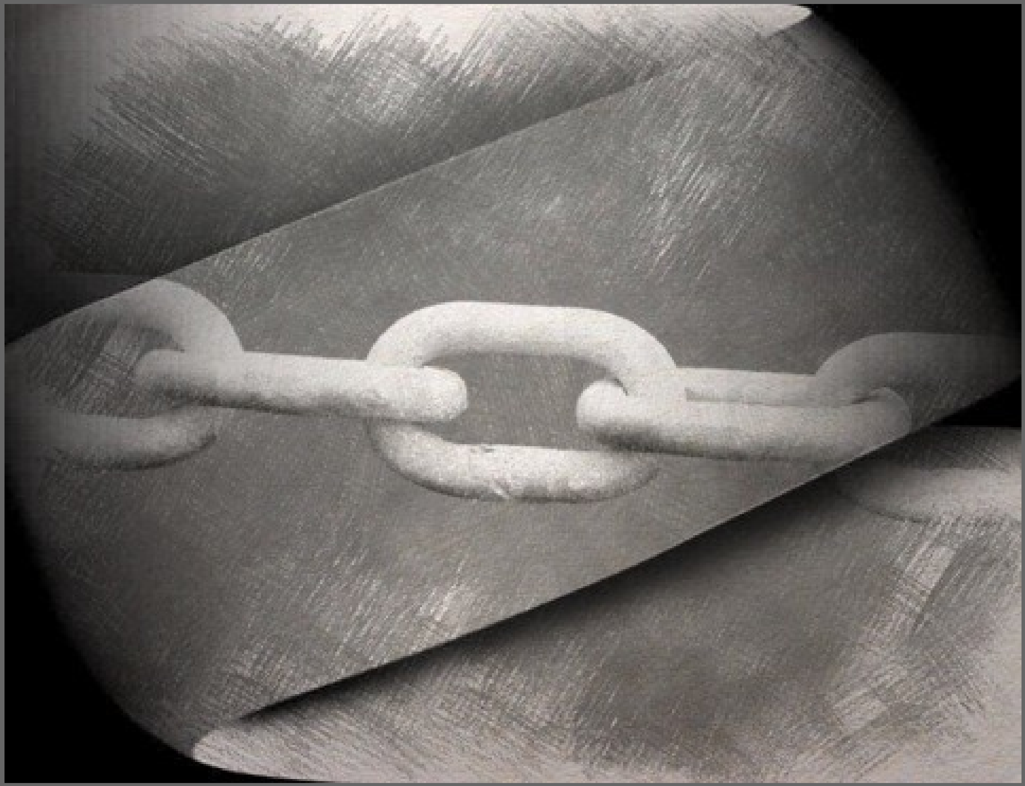

Kettenregel: Aufgaben

a) $f(x) = \dfrac{1}{\sqrt[3]{x}} + \sqrt{2x+1}, \qquad g(x) = \dfrac{1}{x\sqrt[4]{x}} + \sqrt{x^3 + 2x}$

b) $f(x) = (1 + 2\sqrt{x})\,(1 - 2\sqrt{x}), \qquad g(x) = (1 + x^2)\,(1 - x^2)$

c) $f(x) = \sqrt{x}\,\ln(x), \qquad g(x) = (1 - x^2)\,\ln(x - 1)$

d) $f(x) = (4x + 7)^3, \qquad g(x) = (x^3 - 2x)^6$

e) $f(x) = \left(2x + \dfrac{1}{x}\right)^3, \qquad g(x) = \left(x^2 - \dfrac{1}{x^2}\right)^4$

f) $f(x) = (\sqrt{x} + 3x)^4, \qquad g(x) = x^3\,\sqrt{3x - 19}$

Ableitung einer Funktion: Lösung 3 a,b

a) $f(x) = \dfrac{1}{\sqrt[3]{x}} + \sqrt{2x+1} = x^{-\frac{1}{3}} + (2x+1)^{\frac{1}{2}}$

$$f'(x) = -\frac{1}{3x^{4/3}} + \frac{1}{\sqrt{2x+1}} = -\frac{1}{3x\sqrt[3]{x}} + \frac{1}{\sqrt{2x+1}}$$

$$g(x) = \frac{1}{x\sqrt[4]{x}} + \sqrt{x^3+2x} = x^{-\frac{5}{4}} + (x^3+2x)^{\frac{1}{2}}$$

$$g'(x) = -\frac{5}{4x^{9/4}} + \frac{3x^2+2}{2\sqrt{x^3+2x}} = -\frac{5}{4x^2\sqrt[4]{x}} + \frac{3x^2+2}{2\sqrt{x^3+2x}}$$

b) $f(x) = (1+2\sqrt{x})(1-2\sqrt{x}) = 1-4x, \qquad f'(x) = -4$

$$g(x) = (1+x^2)(1-x^2) = 1-x^4, \qquad g'(x) = -4x^3$$

c) $f(x) = \sqrt{x}\,\ln(x), \qquad f'(x) = \frac{1}{2\sqrt{x}}\left(\ln x + 2\right)$

$g(x) = (1 - x^2)\,\ln(x - 1), \qquad g'(x) = -2\,x\ln(x - 1) - x - 1$

d) $f(x) = (4\,x + 7)^3 = 12\,(4\,x + 7)^2$

$g(x) = (x^3 - 2\,x)^6 = 6\,(3\,x^2 - 2)\,(x^3 - 2\,x)^5$

e) $f(x) = \left(2\,x + \frac{1}{x}\right)^3, \qquad f'(x) = 3\left(2\,x + \frac{1}{x}\right)^2\left(2 - \frac{1}{x^2}\right)$

$g(x) = \left(x^2 - \frac{1}{x^2}\right)^4, \qquad g'(x) = 4\left(x^2 - \frac{1}{x^2}\right)^3\left(2\,x + \frac{2}{x^3}\right)$

f) $f(x) = (\sqrt{x} + 3\,x)^4, \qquad f'(x) = 4(\sqrt{x} + 3\,x)^3\left(\frac{1}{2\sqrt{x}} + 3\right)$

$g(x) = x^3\,\sqrt{3\,x - 19}, \qquad g'(x) = 3\,x^2\,\sqrt{3\,x - 19} + \frac{3}{2}\,\frac{x^3}{\sqrt{3\,x - 19}}$

a) $f(x) = \sin(2x - 7)$, $\quad g(x) = \cos(3x^2 + 6x)$

b) $f(x) = \sin(\sqrt{3x - 5})$, $\quad g(x) = \cos(\sqrt{x} + 2x)$

c) $f(x) = \sqrt{\sin x + 2}$, $\quad g(x) = \sqrt{\sin(3x) - 6}$

d) $f(x) = \dfrac{1}{\sin(2x)}$, $\quad g(x) = \dfrac{3}{\sin(x^2 + 1)}$

e) $f(x) = \dfrac{1}{\sin(x^2)}$, $\quad g(x) = \dfrac{1}{\sin(\sqrt{x})}$

f) $f(x) = \sin^2(2x - 1)$, $\quad g(x) = \cos^3(x^2 + 4x)$

g) $f(x) = \sqrt{\sin^2 x + 4}$, $\quad g(x) = \sqrt{\cos^2(3x) + 1}$

a) $f(x) = \sin(2x - 7)$, $\quad f'(x) = 2\cos(2x - 7)$

$g(x) = \cos(3x^2 + 6x)$, $\quad g'(x) = -6(x + 1)\sin(3x^2 + 6x)$

b) $f(x) = \sin(\sqrt{3x - 5})$, $\quad f'(x) = \dfrac{3}{2}\,\dfrac{\cos(\sqrt{3x - 5})}{\sqrt{3x - 5}}$

$g(x) = \cos(\sqrt{x} + 2x)$, $\quad g'(x) = -\left(\dfrac{1}{2\sqrt{x}} + 2\right)\sin(\sqrt{x} + 2x)$

c) $f(x) = \sqrt{\sin x + 2}$, $\quad f'(x) = \dfrac{1}{2}\,\dfrac{\cos x}{\sqrt{\sin x + 2}}$

$g(x) = \sqrt{\sin(3x) - 6}$, $\quad g'(x) = \dfrac{3}{2}\,\dfrac{\cos(3x)}{\sqrt{\sin(3x) - 6}}$

d) $f(x) = \dfrac{1}{\sin(2x)}$, $\quad f'(x) = -\dfrac{2\cos(2x)}{\sin^2(2x)}$

$g(x) = \dfrac{3}{\sin(x^2 + 1)}$, $\quad g'(x) = -\dfrac{6x\cos(x^2 + 1)}{\sin^2(x^2 + 1)}$

e) $f(x) = \dfrac{1}{\sin(x^2)}, \qquad f'(x) = -\dfrac{2x\cos(x^2)}{\sin^2(x^2)}$

$g(x) = \dfrac{1}{\sin(\sqrt{x})}, \qquad g'(x) = -\dfrac{1}{2}\,\dfrac{\cos(\sqrt{x})}{\sqrt{x}\cdot\sin^2(\sqrt{x})}$

f) $f'(x) = 4\sin(2x-1)\cos(2x-1)$

$g'(x) = -6(x+2)\cos^2(x^2+4x)\sin(x^2+4x)$

g) $f(x) = \dfrac{\sin x\,\cos x}{\sqrt{\sin^2 x + 4}} = \dfrac{\sin(2x)}{2\sqrt{\sin^2 x + 4}}$

$g(x) = -\dfrac{3\cos(3x)\sin(3x)}{\sqrt{\cos^2(3x)+1}} = -\dfrac{3\sin(6x)}{2\sqrt{\cos^2(3x)+1}}$